

$$\tau_{\theta} = -k\theta \quad (4)$$

$$\begin{aligned} E_k &= E - E_p = \frac{1}{2} k \theta_{max}^2 - \frac{1}{2} k \theta^2 \quad (5) \\ &= \frac{1}{2} k \theta_{max}^2 - \frac{1}{2} k \left(\frac{\theta_{max}}{\sqrt{2}} \right)^2 \\ &= \frac{1}{2} k \theta_{max}^2 - \frac{1}{2} k \frac{\theta_{max}^2}{2} \\ &= \frac{1}{2} k \theta_{max}^2 - \frac{1}{4} k \theta_{max}^2 \\ &= \frac{1}{4} k \theta_{max}^2 = \frac{1}{2} \times \frac{1}{2} k \theta_{max}^2 \\ &= \frac{1}{2} E \end{aligned}$$

هت منتصف

السؤال الثاني: ص 22 من الكتاب

+ من منتصف ص 23 من الكتاب + ص 24

السؤال الثالث:

$$E = E_p + E_k$$

$$\frac{1}{2} k \theta_{max}^2 = \frac{1}{2} k \theta^2 + \frac{1}{2} I_0 \omega^2$$

نشق الطرفين بمساواة:

$$0 = \frac{1}{2} k 2\theta\theta' + \frac{1}{2} I_0 2\omega\omega'$$

$$0 = k\theta\omega + I_0\omega\alpha$$

$$0 = \omega(k\theta + I_0\alpha)$$

لأنه $\omega \neq 0$

$$k\theta + I_0\alpha = 0 \Rightarrow$$

$$k\theta + I_0(\theta')_t = 0$$

هذا اختبار نواس الفتل

السؤال الأول:

المعطى: $\theta_{max} = 0.8 \text{ rad}$

$$\frac{1}{2} T_0 = 2 \Rightarrow T_0 = 4 \text{ s} \Rightarrow$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad.s}^{-1}$$

منه φ من شرط الجيب:

$$t=0 \left\{ \Rightarrow \bar{\theta} = \theta_{max} \cos(\omega_0 t + \varphi) \right.$$

$$\theta = \theta_{max} \left. \begin{aligned} &\Rightarrow \theta_{max} = \theta_{max} \cos \varphi \Rightarrow \\ &\cos \varphi = 1 \Rightarrow \varphi = 0 \text{ rad} \end{aligned} \right\}$$

$$\Rightarrow \bar{\theta} = 0.8 \cos \frac{\pi}{2} t$$

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}} \quad (2)$$

$$T_0' = 2\pi \sqrt{\frac{4I_0}{k}} = 2 \times 2\pi \sqrt{\frac{I_0}{k}}$$

$$T_0' = 2 T_0 = 2 \times 2 = 4 \text{ s}$$

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}} \quad k = k' \frac{(2r)^4}{l} \quad (3)$$

لأنه عند زيادة طول السلك إلى نصف

$$k^* = \frac{k' (2r)^4}{2l} = \frac{k}{2} \Rightarrow$$

$$T_0' = 2\pi \sqrt{\frac{I_0}{k^*}} = 2\pi \sqrt{\frac{I_0}{\frac{k}{2}}}$$

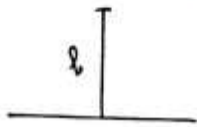
$$T_0' = 2\pi \sqrt{\frac{2I_0}{k}} = \sqrt{2} \times 2\pi \sqrt{\frac{I_0}{k}}$$

$$T_0' = \sqrt{2} T_0 = \sqrt{2} \times 0.5 = \frac{\sqrt{2}}{2} \text{ s}$$

$$\frac{1}{k_1} = \frac{4}{k_2} \Rightarrow k_2 = 4k_1$$

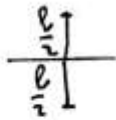
$$T_{02} = 2\pi \sqrt{\frac{I_0}{k_2}} = 2\pi \sqrt{\frac{I_0}{4k_1}} \quad \text{لأنه}$$

$$T_{02} = \frac{1}{2} \times 2\pi \sqrt{\frac{I_0}{k_1}} = \frac{1}{2} T_{01}$$



السؤال الخامس: (1)

$$k = k' \frac{(2r)^4}{l} \quad T_0 = 2\pi \sqrt{\frac{I_0}{k}}$$



$$k^* = k_1 + k_2 = k' \frac{(2r)^4}{\frac{l}{2}} + k' \frac{(2r)^4}{\frac{l}{2}}$$

$$k^* = 2k' \frac{(2r)^4}{l} + 2k' \frac{(2r)^4}{l}$$

$$k^* = 2k + 2k = 4k$$

$$\Rightarrow T_0' = 2\pi \sqrt{\frac{I_0}{k^*}} = 2\pi \sqrt{\frac{I_0}{4k}}$$

$$T_0' = \frac{1}{2} \times 2\pi \sqrt{\frac{I_0}{k}} = \frac{1}{2} T_0$$

$$k = k' \frac{(2r)^4}{l} \quad T_0 = 2\pi \sqrt{\frac{I_0}{k}} \quad \text{لأنه}$$

$$k^* = k' \frac{(2r)^4}{\frac{l}{4}} = 4k' \frac{(2r)^4}{l} = 4k$$

$$T_0' = 2\pi \sqrt{\frac{I_0}{k^*}} = 2\pi \sqrt{\frac{I_0}{4k}} = \frac{1}{2} \times 2\pi \sqrt{\frac{I_0}{k}} \quad T_0' = \frac{1}{2} T_0$$

$$(\ddot{\theta})_t = -\frac{k}{I_0} \bar{\theta} \quad \dots (1)$$

وهي معادلة تفاضلية من الدرجة الثانية
تقبل حلاً جيبياً من الشكل:

$$\bar{\theta} = \theta_{\max} \cos(\omega_0 t + \bar{\varphi}) \quad \dots (2)$$

للتأكد من أنه للمعادلة (2) حل للمعادلة (1)
نشتق المعادلة (2) مرتين بالنسبة للزمن

$$(\dot{\theta})_t = -\omega_0 \theta_{\max} \sin(\omega_0 t + \bar{\varphi})$$

$$(\ddot{\theta})_t = -\omega_0^2 \theta_{\max} \cos(\omega_0 t + \bar{\varphi})$$

$$(\ddot{\theta})_t = -\omega_0^2 \bar{\theta} \quad \dots (3)$$

مباراة (1) و (3) نجد أنه:

$$\omega_0 = \sqrt{\frac{k}{I_0}} > 0$$

حيث k و I_0 موجبتين، ومنه

$$\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{k}{I_0}} \Rightarrow T_0 = 2\pi \sqrt{\frac{I_0}{k}}$$

وهي فترة اهتزاز جيبية توافقية بسيطة.

السؤال الرابع:

$$k_1 = k' \frac{(2r)^4}{l_1}$$

$$k_2 = k' \frac{(2r)^4}{l_2}$$

$$\Rightarrow l_1 = k' \frac{(2r)^4}{k_1}$$

$$\Rightarrow l_2 = k' \frac{(2r)^4}{k_2}$$

$$l_1 = 4l_2$$

لأنه

$$\frac{k' \cdot (2r)^4}{k_1} = 4 \frac{k' \cdot (2r)^4}{k_2}$$

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بدون وجود كتلتين (4)

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}}$$

بوجود كتلتين

$$T_0' = 2\pi \sqrt{\frac{I_0 + I_{01}m_1 + I_{02}m_2}{k}}$$

$$T_0' = 2\pi \sqrt{\frac{I_0 + m_1r_1^2 + m_2r_2^2}{k}}$$

$$T_0' = 2\pi \sqrt{\frac{I_0 + 2m_1r_1^2}{k}}$$

$$\frac{T_0'}{T_0} = \frac{2\pi \sqrt{\frac{I_0 + 2m_1r_1^2}{k}}}{2\pi \sqrt{\frac{I_0}{k}}}$$

$$\frac{T_0'}{T_0} = \sqrt{\frac{I_0 + 2m_1r_1^2}{I_0}}$$

$$T_0' = T_0 \sqrt{1 + \frac{2m_1r_1^2}{I_0}} \quad r = \frac{l}{2}$$

$$T_0' = T_0 \sqrt{1 + \frac{2 \times 20 \times 10^{-3} \times (\frac{1}{2})^2}{2 \times 10^{-3}}}$$

$$T_0' = 1 \sqrt{1 + 5}$$

$$T_0' = \sqrt{6} \text{ s}$$

ما k :

$$k = \omega_0^2 I_0 = \left(\frac{2\pi}{T_0}\right)^2 \times I_0$$

$$k = 40 \times 2 \times 10^{-3} = 8 \times 10^{-2} \text{ mV rad}^{-2}$$

المعادلة (1):
 المعادلة (2):

$$\bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1} = 2\pi \text{ rad} \cdot \text{s}^{-1}$$

$$\left. \begin{matrix} t=0 \\ \omega=0 \end{matrix} \right\} \Rightarrow \theta = \theta_{max} = \frac{\pi}{2} \text{ rad}$$

حسب $\bar{\varphi}$ من شرط البداية:

$$\left. \begin{matrix} t=0 \\ \theta = \theta_{max} \end{matrix} \right\} \Rightarrow \bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{max} = \theta_{max} \cos \bar{\varphi} \Rightarrow$$

$$\cos \bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0 \text{ rad}$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{2} \cos(2\pi t)$$

(2)

$$\omega = (\dot{\theta})_t = -\omega_0 \theta_{max} \sin(\omega_0 t + \bar{\varphi})$$

$$\omega = -2\pi \times \frac{\pi}{2} \sin(2\pi t)$$

$$\omega = -10 \sin 2\pi t$$

لأنه لحظة البدء الزاوية بدوئتها

$$t = \frac{T_0}{4} = \frac{1}{4} \text{ s} \Rightarrow$$

$$\omega = -10 \sin \frac{2\pi}{4} = -10 \sin \frac{\pi}{2}$$

$$\omega = -10 \text{ rad} \cdot \text{s}^{-1}$$

(3)

$$\alpha = -\omega_0^2 \theta = -(2\pi)^2 \times -\frac{\pi}{4}$$

$$\alpha = +40 \times \frac{\pi}{4} = 10\pi \text{ rad} \cdot \text{s}^{-2}$$

$$I_{O/C} = \frac{1}{2} m r^2 \Rightarrow$$

$$m = \frac{2I_0}{r^2} = \frac{2 \times 2 \times 10^{-3}}{(0.2)^2}$$

$$m = 0.1 \text{ kg}$$

المعادلة الثالثة:

$$K = \omega_0^2 I_0 \dots (1)$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi \text{ rad.s}^{-1}$$

$$I_0 = I_{O1} m_1 + I_{O2} m_2$$

$$= m_1 r_1^2 + m_2 r_2^2 = 2 m_1 r_1^2 \quad r = \frac{l}{2}$$

$$= 2 \times 100 \times 10^{-3} \times (20 \times 10^{-2})^2$$

$$= 0.2 \times 400 \times 10^{-4} = 8 \times 10^{-3} \text{ kg.m}^2$$

نعوض في (1):

$$K = (\pi)^2 \times 8 \times 10^{-3} = 8 \times 10^{-2} \text{ mN.rad}^{-1}$$

$$\bar{\theta} = \theta_{\max} \cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi \text{ rad.s}^{-1}$$

$$t=0 \left. \begin{array}{l} \\ \omega=0 \end{array} \right\} \Rightarrow \theta = \theta_{\max} = \frac{\pi}{3} \text{ rad}$$

نستعمل شرط البداية:

$$t=0 \left. \begin{array}{l} \\ \theta = \theta_{\max} \end{array} \right\} \Rightarrow \bar{\theta} = \theta_{\max} \cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{\max} = \theta_{\max} \cos \bar{\varphi}$$

(3)

$$E = \frac{1}{2} K \theta_{\max}^2 = \frac{1}{2} \times 8 \times 10^{-2} \left(\frac{\pi}{3}\right)^2$$

$$E = 0.2 \text{ J}$$

المعادلة الثانية: (1)

$$\theta = \theta_{\max} \cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \sqrt{\frac{K}{I_0}} = \sqrt{\frac{8 \times 10^{-2}}{2 \times 10^{-3}}} = 2\pi \text{ rad.s}^{-1}$$

$$t=0 \left. \begin{array}{l} \\ \omega=0 \end{array} \right\} \Rightarrow \theta = \theta_{\max} = \frac{\pi}{2} \text{ rad}$$

نستعمل شرط البداية:

$$t=0 \left. \begin{array}{l} \\ \theta = \theta_{\max} \end{array} \right\} \Rightarrow \bar{\theta} = \theta_{\max} \cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{\max} = \theta_{\max} \cos \bar{\varphi} \Rightarrow$$

$$\cos \bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0 \text{ rad}$$

$$\Rightarrow \bar{\theta} = 2\pi \cos \frac{\pi}{2} t$$

$$E_p = \frac{1}{2} K \theta^2 = \frac{1}{2} \times 8 \times 10^{-2} \left(\frac{\pi}{6}\right)^2$$

$$= 4 \times 10^{-2} \times \frac{10}{36} = \frac{1}{90} \text{ J}$$

$$E = \frac{1}{2} K \theta_{\max}^2 = \frac{1}{2} \times 8 \times 10^{-2} \times \left(\frac{\pi}{2}\right)^2$$

$$E = \frac{1}{10} \text{ J}$$

$$\Rightarrow E_k = E - E_p = \frac{1}{10} - \frac{1}{90}$$

$$E_k = \frac{8}{90} \text{ J}$$

(1)

(2)

(2)

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$$T_0 = 2\pi \sqrt{\frac{12.9 \times 10^{-6}}{9 \times 10^4}}$$

$$T_0 = \pi \text{ s}$$

$$T_0' = T_0 = 1.4 = 3.14 = 1.14$$

$$T_0' = 2 \text{ s}$$

$$T_0' = 2\pi \sqrt{\frac{I_0}{k}} = 2\pi \sqrt{\frac{I_0}{\frac{2m_1 r^2}{2}}}$$

$$2 = 2\pi \sqrt{\frac{2.9 \times 10^{-6} + 2 \times 20 \times 10^{-3} r^2}{5 \times 10^4}}$$

نربح الطرفين

$$4 = 40 \frac{2.9 \times 10^{-6} + 4 \times 10^{-2} r^2}{5 \times 10^4}$$

$$\frac{4 \times 5 \times 10^{-4}}{40} = 2.9 \times 10^{-6} + 4 \times 10^{-2} r^2$$

$$5 \times 10^{-5} - 2.9 \times 10^{-6} = 4 \times 10^{-2} r^2$$

$$2.9 \times 10^{-5} = 4 \times 10^{-2} r^2$$

$$r^2 = \frac{2.9 \times 10^{-5}}{4 \times 10^{-2}} = 6.29 \times 10^{-6}$$

$$r = 2.9 \times 10^{-3} \text{ m}$$

$$\cos \bar{\alpha} = 1 \Rightarrow \bar{\alpha} = 0 \text{ rad}$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{3} \cos(\pi t)$$

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}} \quad k = k' \frac{(2r)^4}{l} \quad (3)$$

$$T_0' = 2\pi \sqrt{\frac{I_0}{k^*}} \quad \frac{l}{2}$$

$$k^* = k' \frac{(2r)^4}{\frac{l}{2}} = 2 k' \frac{(2r)^4}{l} = 2k$$

$$\Rightarrow T_0' = 2\pi \sqrt{\frac{I_0}{2k}} = \frac{1}{\sqrt{2}} \times 2\pi \sqrt{\frac{I_0}{k}}$$

$$T_0' = \frac{T_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} \text{ s}$$

المعادلة السابقة (1)

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}}$$

$$I_0 = I_0 + I_{01m_1} + I_{01m_2}$$

$$= \frac{1}{2} M_1 R^2 + 2 m_1 R^2$$

$$= \frac{1}{2} (0.2) (0.05)^2 + 2 (20 \times 10^{-3}) (0.05)^2$$

$$= 0.01 \times 2.9 \times 10^{-4} + 4 \times 10^{-2} \times 2.9 \times 10^{-4}$$

$$= 2.9 \times 10^{-6} + 100 \times 10^{-6}$$

$$= 129 \times 10^{-6} \text{ Kg m}^2$$

المسألة الخامسة: (1)

$$\bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad.s}^{-1}$$

$$\left. \begin{matrix} t=0 \\ \omega=0 \end{matrix} \right\} \Rightarrow \theta = \theta_{max} = \frac{\pi}{3} \text{ rad}$$

عند $\bar{\varphi}$ شرط الجيب:

$$\left. \begin{matrix} t=0 \\ \theta = \theta_{max} \end{matrix} \right\} \Rightarrow \bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{max} = \theta_{max} \cos \bar{\varphi}$$

$$\cos \bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0 \text{ rad}$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{3} \cos\left(\frac{\pi}{2} t\right)$$

(2)

$$T_0 = 2\pi \sqrt{\frac{I_0}{k}}$$

$$T_0 = 2\pi \sqrt{\frac{I_{01} + I_{02}}{k}}$$

$$T_0 = 2\pi \sqrt{\frac{2m_1 r^2}{k}} \quad r = \frac{l}{2}$$

$$T_0 = 2\pi \sqrt{\frac{2m_1 \frac{l^2}{4}}{k}}$$

نرتب المعادلة

$$T_0^2 = 40 \frac{m_1 \frac{l^2}{4}}{k} \Rightarrow$$

$$\frac{l^2}{4} = \frac{T_0^2 \times k}{40m} \Rightarrow l^2 = \frac{T_0^2 \times k}{20m}$$

$$l = T_0 \sqrt{\frac{k}{20m}} = 4 \sqrt{\frac{8 \times 10^4}{20 \times 20 \times 10^{-3}}}$$

$$l = 4 \sqrt{\frac{8}{4000}} = 4 \sqrt{\frac{1}{500}}$$

$$l = \frac{4}{10\sqrt{5}} = \frac{2}{5\sqrt{5}} \text{ m}$$

(3)

$$\omega = -\omega_0 \theta_{max} \sin(\omega_0 t + \bar{\varphi})$$

$$\omega = -\frac{\pi}{2} \times \frac{\pi}{3} \sin\left(\frac{\pi}{2} t\right)$$

$$\omega = -\frac{10}{6} \sin\frac{\pi}{2} t = -\frac{5}{3} \sin\frac{\pi}{2} t$$

لحظة التوازن يكون $t = \frac{3T_0}{4} = \frac{3 \times 4}{4} = 3 \text{ s}$

$$\Rightarrow \omega = -\frac{5}{3} \sin\frac{3\pi}{2}$$

$$\omega = +\frac{5}{3} \text{ rad.s}^{-1}$$