Maths

EL-MOASSER

2023

Model Exams

According to the specifications of the examination paper

3rd PREP.

FIRST TERM

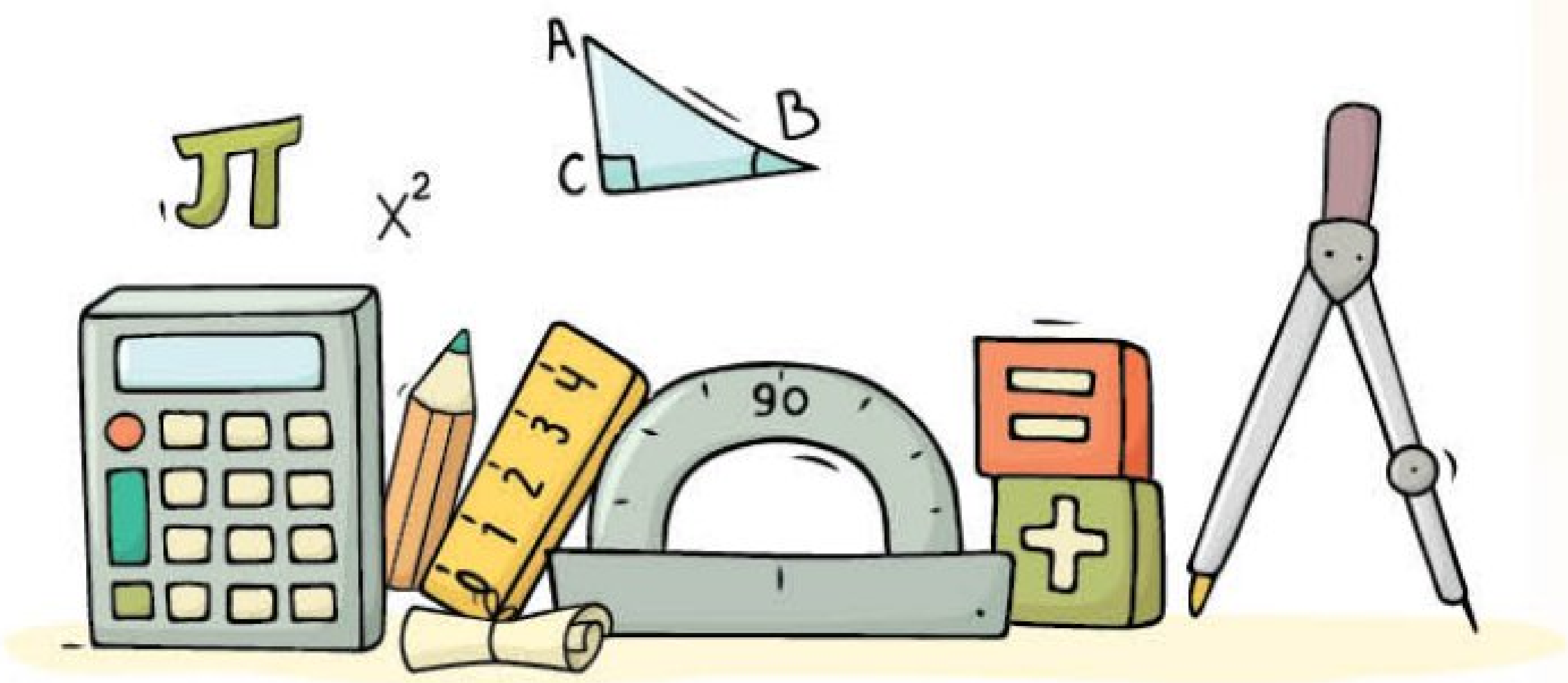
Table of specifications of the examination paper

In **Algebra and Statistics - First Term**

3rd prep.

Contents	The total number of questions	Marks	
		Marks	Total
Accumulative basic skills	(3) Objective items	3	3
The Cartesian product - Relation - Functions - Polynomial Functions	(1) Objective item	1	10
	$(1\frac{1}{2})$ Essay question	9	
The ratio and proportion - The direct variation and inverse variation	(1) Objective item	1	13
	(2) Essay questions	12	
Statistics	(1) Objective item	1	4
	$(\frac{1}{2})$ Essay question	3	
Total	(1) Objective question	6	30
	(4) Essay questions	24	

* **The examination paper consists of 5 questions :**
  - **The first question is multiple choice question of 6 items out of 6 marks**
  - **Four eassy questions each out of 6 marks**
* **The total mark : 30 marks**
* **Time allowed : Two hours**



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 If $2a + 2b + c = 36$ and $a + b = 15$, then $c =$ ....

(a) 3 (b) 6 (c) 10 (d) 21

2 If $\sum (X - \overline{X})^2 = 36$ of a set of values and number of these values is 9 , then the standard deviation = ...

(a) 2 (b) 18 (c) 27 (d) 4

3 If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ ....

(a) X (b) Y (c) X × Y (d) Z

4 $\{7\} \subset$ ....

(a) (5 , 7) (b) $\{17 , 2\}$ (c) $]3 , 7[$ (d) $\{3 , 7\}$

5 Which of the following is the closest to $11^2 + 9^2$?

(a) 20 + 20 (b) 20 + 80 (c) 120 + 20 (d) 120 + 80

6 If $y^2 + 4X^2 = 4Xy$, then : ....

(a) $y \propto X$ (b) $y \propto X^2$ (c) $y \propto \frac{1}{X}$ (d) $y \propto \frac{1}{X^2}$

2 **[a]** If $X = \{0 , 1 , 4 , 7\}$, $Y = \{1 , 3 , 5 , 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 6" for all $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and show if R is a function or not and why ?

[b] If $f(X) = 2X - 1$, **then prove that :** $f(2) - 3f(1) =$ zero

3 **[a]** If y varies directly as X and $y = \frac{5}{3}$ when $X = \frac{1}{6}$, find the value of X when $y = \frac{3}{4}$

[b] Find the number that if we subtract it from each of the numbers 3 , 7 , 19 , then they become in continued proportion.

4 **[a]** If $\frac{X}{y} = \frac{2}{3}$, **find the value of the ratio :** $\frac{3X + 2y}{6y - X}$

[b] If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$, **prove that :** $\frac{a - b + c}{a + b - c} = \frac{1}{3}$

5 **[a]** Represent graphically the function $f : f(X) = X^2 - 4X + 3$ taking $X \in [-1, 5]$

and from the graph deduce :

1. The maximum or minimum value of the function f
2. The equation of the axis of symmetry

[b] Calculate the mean and the standard deviation of the following data :

2 , 3 , 6 , 8 , 11

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If $X = \{1, 2\}$, $Y = \{0\}$, then $n(X \times Y) =$ ....
   (a) zero (b) 1 (c) 2 (d) 3
2. $2^5 + 2^5 + 2^5 + 2^5 =$ ....
   (a) 2^4 (b) 2^6 (c) 2^7 (d) 2^{20}
3. Double of the number 2^{19} is ....
   (a) 2^{38} (b) 2^{20} (c) 2^{30} (d) 4^{19}
4. (The greatest value – the smallest value) of the set of data is ....
   (a) the median. (b) the range.
   (c) the mode. (d) the standard deviation.
5. $\mathbb{R}^+ \cup \mathbb{R}^- =$ ....
   (a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R}^*$ (d) $\{0\}$
6. The middle proportional between 4 and 36 is ....
   (a) 32 (b) 40 (c) 12 (d) ± 12

2 **[a]** If $X = \{1, 2, 3\}$, $Y = \{1, 3, 4, 6, 9\}$ and R is a relation from X ot Y where "a R b" means "b = 3 a" for all $a \in X$, $b \in y$

1. Write R
2. Represent it by an arrow diagram
3. Show if R is a function or not and why ?

[b] Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 , it becomes 3 : 5

3 **[a]** If $\frac{X + y}{5} = \frac{y + z}{3} = \frac{z + X}{6}$, **prove that :** $\frac{X - z}{2} = \frac{X + y + z}{7}$

[b] Represent graphically the function $f : f(X) = (X - 3)^2$, taking $X \in [0, 6]$ and from the graph deduce the coordinates of the vertex of the curve and the minimum or maximum value of the function.

4 **[a]** If y is the middle proportional between x and z , **prove that :** $\frac{x z}{y (y + z)} = \frac{x}{x + y}$

[b] If y = 1 + b where b is varies inversely with the square of x and if : y = 17 when $x = \frac{1}{2}$, then find the relation between y and x , then find the value of y when $x = 2$

5 **[a]** If $X \times Y = \{(1 , 1) , (1 , 2) , (1 , 3)\}$

Find : 1 X , Y   2 $Y \times X$ 3 X^2

[b] Calculate the standard deviation to the following data :

data	0	1	2	3	4	5
Frequency	9	15	17	25	20	14

Model 3

Answer the following questions :

1 **Choose the correct answer from those given :**

1 Which of the following relations represents an inverse variation between the two variables X and Y ?

(a) $y^2 = 2 x$ (b) $\frac{y}{x} - x = 2$ (c) $x y = 3$ (d) $\frac{y}{x} - 9 = 5$

2 The set which has more dispersion of the following sets is ....

(a) 28 , 17 , 30 , 36 , 20   (b) 31 , 35 , 26 , 37 , 41

(c) 25 , 39 , 19 , 5 , 27   (d) 20 , 19 , 29 , 37 , 43

3 If M represent a negative number , which of the following represents a positive number ?

(a) M^2 (b) M^3 (c) 2M   (d) $\frac{M}{2}$

4 If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then : x = ...

(a) 8   (b) 6   (c) 3   (d) 5

5 Quarter of the number 4^{20} is ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

6 The next number in the pattern : $\frac{1}{1000} , \frac{1}{100} , \frac{1}{10}$, ... is ....

(a) zero   (b) 1   (c) 10   (d) 100

2 **[a]** If R is a relation on $\mathbb{N}$ (the set of natural numbers) where a R b means "a × b = 18" for all $a \in \mathbb{N}$, $b \in \mathbb{N}$, write R and represent it by an arrow diagram.

[b] If b is the middle proportional between a and c , **prove that :** $\frac{2 c^2 - 3 b^2}{2 b^2 - 3 a^2} = \frac{c}{a}$

3 **[a]** If $y \propto \frac{1}{X}$ and if y = 2 when $X = 6$

Find : 1 The Relation between X and y   2 The value of X when y = 8

[b] If $X = \{2, 3, 4\}$, $Y = \{3, 4, 5, 6, 7, 8\}$ and If $f : X \longrightarrow Y$ where $f(X) = 9 - X$

Find the images of the elements of the set X by the function f

4 **[a]** Find the number that if we added it to each of the numbers 1 , 7 , 25 , then they become in continued proportion.

[b] Represent graphically the function $f : f(X) = X^2 - 4$ where $X \in [-3, 3]$

From the graph deduce :

1 The equation of the axis of symmetry.   2 The minimum value of the function.

5 **[a]** If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$ **Find :** the value of X

[b] Calculate the mean and the standard deviation for the following frequency distribution :

Sets	0 –	2 –	4 –	6 –	8 –
Frequency	5	9	15	15	6

Model 4

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\mathbb{Z}^- \cup \mathbb{N} =$ ……………

(a) $\varnothing$ (b) $\mathbb{N}$ (c) $\mathbb{Z}$ (d) $\mathbb{R}$

2 If $\frac{3a}{5b} = \frac{1}{2}$, then $\frac{a}{b} =$ ……………

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

3 $\left(\frac{\sqrt{5}+1}{2}\right)^{1000} \left(\frac{\sqrt{5}-1}{2}\right)^{1000} =$ ……………

(a) zero   (b) 1   (c) $\frac{5^{1000}-1}{4}$ (d) 4^{1000}

4 If $f(X) = kX + 8$, $f(2) = 0$, then k = ……………

(a) 8   (b) 6   (c) 4   (d) – 4

5 The ratio between the area of a square shaped region of side length X cm. to the area of another square shaped region of side length $2X$ cm. as a ratio ……………

(a) 1 : 2   (b) X : 4   (c) 1 : 4   (d) 4 : 1

6 If all individuals are equal in values , then ……………

(a) $X - \overline{X} > 0$ (b) $X - \overline{X} < 0$ (c) $\sigma = 0$ (d) $\overline{X} = 0$

2 **[a]** Find the number that if it is added to each of the numbers 3 , 5 , 8 and 12 , they become proportional.

[b] If $\frac{21 x - y}{7 x - z} = \frac{y}{z}$, **prove that :** $y \propto z$

3 **[a]** Represent graphically the quadratic function $f : f(x) = x^2 - 2$ taking $x \in [-3, 3]$ and from the graph deduce :

1. The vertex of the curve.
2. The minimum of value of the function.
3. The equation of the axis of symmetry.

[b] If $f(x) = x^2 - 3x$, $g(x) = x - 3$

1. **Find :** $f(\sqrt{2}) + 3\, g(\sqrt{2})$
2. **Prove that :** $f(3) = g(3) = 0$

4 **[a] If a , b , c and d are in continued proportion , prove that :** $\frac{c^2 - d^2}{a - c} = \frac{b\,d}{a}$

[b] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y , where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

5 **[a]** If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **prove that :** $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

[b] The following frequency distribution shows the ages of 10 children :

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

Model 5

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The next number in the pattern : $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ is ....

(a) $\sqrt{50}$ (b) $\sqrt{75}$ (c) $\sqrt{60}$ (d) $\sqrt{90}$

2. If $(x^3, y^2) = (1, 4)$, $x > y$, then $x\,y =$ ....

(a) 4 (b) 2 (c) – 2 (d) – 4

3. Four times the number $2^8 =$ ....

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

4. If the standard deviation of a set of data : $x + 2$, 5 , $y - 2$ equals zero , then $x + y =$ ....

(a) 4 (b) 5 (c) 9 (d) 10

5 If 7 , X and $\frac{1}{y}$ are in continued proportion , then X^2 y = ...

(a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 49

6 The sum of the integers in the interval $[-5 , 5[$ is ....

(a) zero (b) 10 (c) – 5 (d) 5

2 [a] If $f(X) = a X + b$, $f(a) = b$, **find the value of :** $a b^2 + 5$

[b] If a , b , c , and d are proportional quantities , **prove that :** $\sqrt[3]{\frac{5 a^3 - 3 c^3}{5 b^3 - 3 d^3}} = \frac{a + c}{b + d}$

3 **[a]** If X = {1 , 2 , 3} , Y = {1 , 3 , 6 , 9 , 12} and R is a relation from X to Y , where "a R b" means "a = $\frac{1}{3}$ b" for each a ∈ X , b ∈ Y

Write R and show that it is a function and write its range.

[b] If y = a – 9 and y ∝ $\frac{1}{X^2}$ and a = 18 when $X = \frac{2}{3}$, find the relation between y and X , then deduce the value of y when $X = 1$

4 **[a]** If $f(X) = a + X^2$, $\ell(X) = c$ are two polynomial functions where $3 f(2) + 3 \ell(X) = 6$, find the numerical value of : $2 f(0) + 2 \ell(7)$ where a and c are constants.

[b] If $\frac{X}{y} = \frac{2}{3}$, **find the value of the ratio :** $\frac{3 X + 2 y}{6 y - X}$

5 **[a] If b is the middle proportional between a and c , prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] The following are the frequency distribution for a number of defective units found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the defective units.

Table of specifications of the examination paper

In **Geometry and Trigonometry - First Term**

3rd prep.

Contents	The total number of questions	Marks	
		Marks	Total
Accumulative basic skills	(3) Objective items	3	3
Trigonometry	(1) Objective item	1	10
	($1\frac{1}{2}$) Essay questions	9	
Analytical geometry	(2) Objective items	2	17
	($2\frac{1}{2}$) Essay questions	15	
Total	(1) Objective question	6	30
	(4) Essay questions	24	

* **The examination paper consists of 5 questions :**
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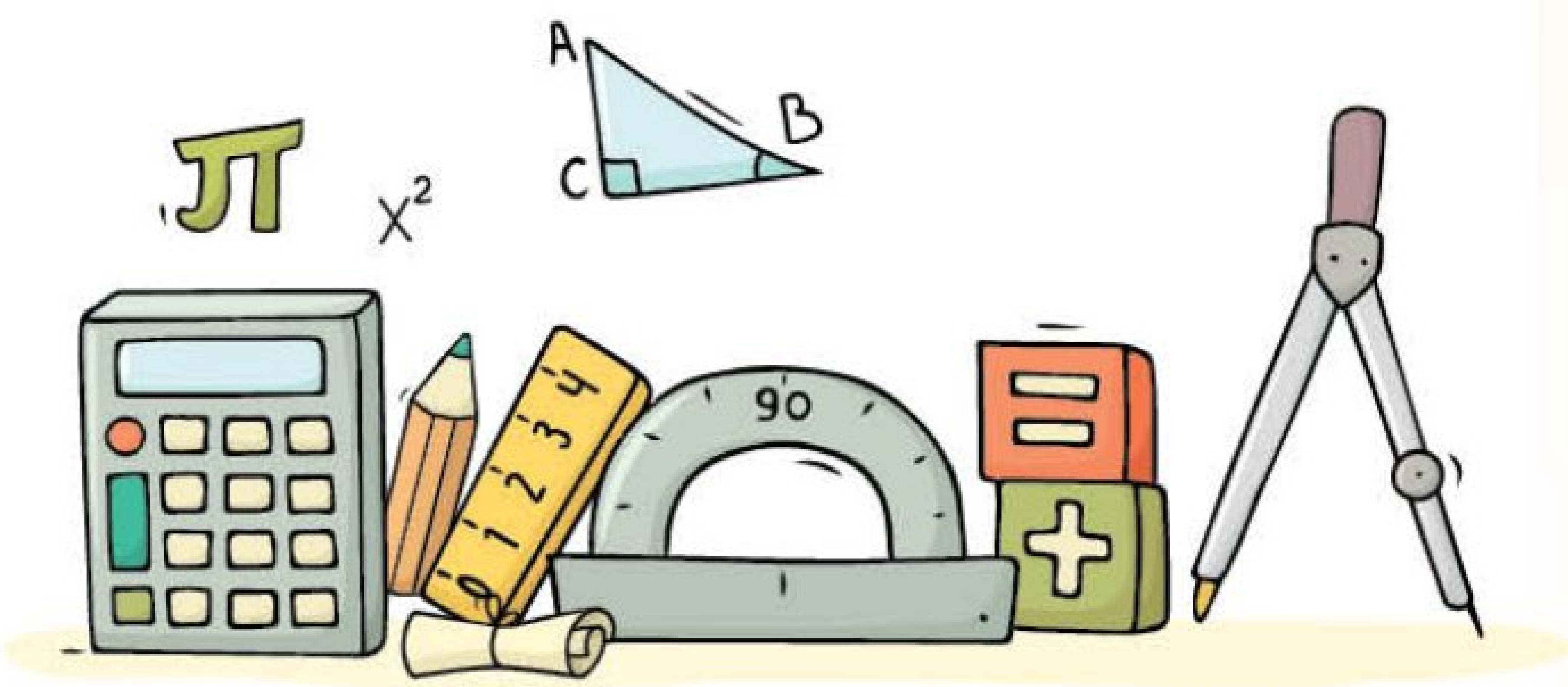


Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 If the two straight lines : $X + 3y - 6 = 0$ and $aX - 3y + 7 = 0$ are perpendicular , then a = ...

(a) 2 (b) 9 (c) 4 (d) 1

2 The sum of the measures of the accumulative angles at a point equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

3 If 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ may be equal to ....

(a) 3 (b) 4 (c) 7 (d) 10

4 The points (– 3 , 0) , (0 , 3) and (3 , 0) are the vertices of .... triangle.

(a) a scalene (b) an equilateral

(c) an obtuse (d) a right-angled and isosceles

5 If $\sin (X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is an acute angle , then X = ...

(a) 30° (b) 60° (c) 25° (d) 55°

6 The image of the point (– 3 , 5) by moving 3 units in the negative direction of the y-axis is ....

(a) (– 3 , 8) (b) (– 3 , 2) (c) (– 6 , 5) (d) (0 , 5)

2 **[a] Prove that :** Δ ABC where A (– 3 , 0) , B (3 , 4) and C (1 , – 6) is an isosceles triangle.

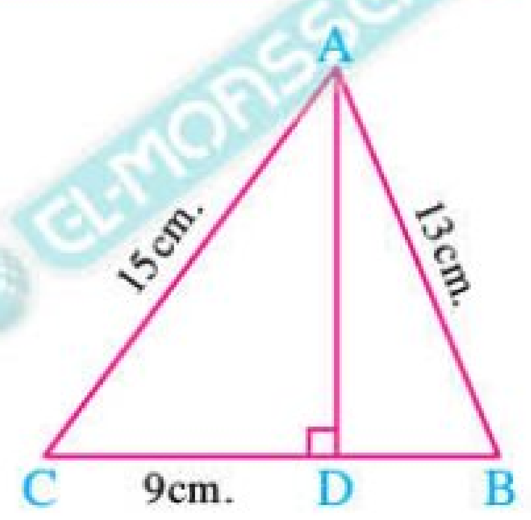
[b] Find the equation of the straight line that passes through the point (1 , 3) and perpendicular to the line whose equation is : $y = \frac{1}{4} X - 2$

3 **[a] In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, AB = 13 cm. , AC = 15 cm. , CD = 9 cm.

Find in simplest form the value of :

$$\frac{\tan (\angle CAD) + \tan (\angle BAD)}{\tan (\angle CAD) - \tan (\angle BAD)}$$



[b] ABCD is a parallelogram where E is the intersection point of its diagonals where A (3 , – 1) , B (6 , 2) and C (1 , 7)

Find : 1 The coordinates of E and D 2 The length of $\overline{DE}$

4 **[a] Prove that :** $\tan 60° = 2 \tan 30° \div (1 - \tan^2 30°)$

[b] Find the equation of the axis of symmetry of $\overline{XY}$, where X (3 , – 2) and Y (– 5 , 6)

5 **[a]** If $\sin E = \sin 60° \cos 30° - \sin 30° \cos 60°$, Find without using the calculator the value of m (∠ E) where E is an acute angle.

[b] Prove that : The points A (4 , 3) , B (1 , 1) and C (– 5 , – 3) are collinear.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $\cos 2X = \frac{1}{2}$ where $2X$ is an acute angle , then $X =$ ……………

(a) 15° (b) 30° (c) 45° (d) 60°

2 A circle its centre is the origin point and its radius length is 3 length units , then the point …………… belongs to it.

(a) (1 , 2) (b) $(-2 , \sqrt{5})$ (c) $(\sqrt{3} , 1)$ (d) $(\sqrt{2} , 1)$

3 In Δ ABC , if m (∠ B) > m (∠ C) , then ……………

(a) AC – AB < 0 (b) AC – AB ≤ 0 (c) BC ≤ AB (d) AC – AB > 0

4 The number of diagonals of a hexagon is ……………

(a) 6 (b) 3 (c) 12 (d) 9

5 If the point (0 , 4) bisects the distance between the two points (– 1 , – 1) and (X , y) , then the point (X , y) is ……………

(a) (1 , 9) (b) (– 1 , 9) (c) $\left(\frac{-1}{2} , \frac{3}{2}\right)$ (d) (– 1 , 3)

6 If the lengths of two sides of an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side equal ……………

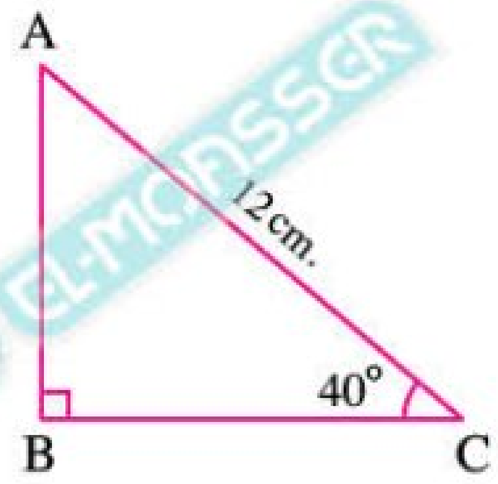
(a) 2 cm. (b) 3 cm. (c) 5 cm. (d) 7 cm.

2 **[a]** Find the equation of the line passing through the point (1 , 6) and the midpoint of $\overline{AB}$ where A (1 , – 2) and B (3 , – 4)

[b] Prove that : $\sin^3 30° = 9 \cos^3 60° - \tan^2 45°$

3 **[a]** If a triangle whose vertices are A (3 , – 1) , B (X , 3) and C (5 , 3) is right-angled at A , **find :** The value of X , then find its area.

[b] In the opposite figure :



m (∠ B) = 90° , m (∠ C) = 40°

, AC = 12 cm

Find : 1 The length of $\overline{AB}$

2 The length of $\overline{BC}$ to the nearest cm.

4 **[a] Without using calculator , prove that :** $\frac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°} = \tan^2 45°$

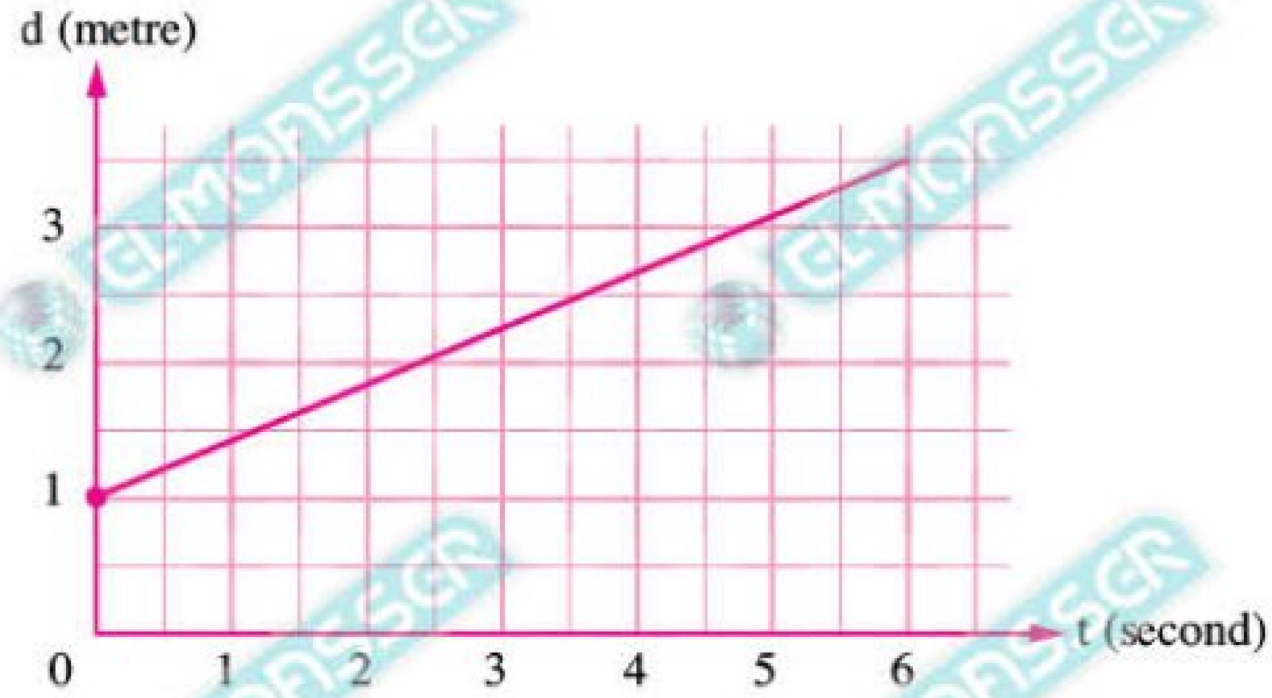
[b] $\overline{AB}$ is a diameter in a circle of centre M , if B (8 , 11) and M (5 , 7)

Find : 1 The coordinates of the point A

2 The radius length of the circle.

3 Equation of a straight line perpendicular to $\overline{AB}$ from a point B

5 The opposite graph represents the motion of a particle moving with uniform velocity (v) where the distance (d) is measured in metre and the time (t) in seconds.



Find :

1 The distance at the beginning of the motion.

2 The velocity of the particle.

3 The equation of the straight line representing the motion of the particle.

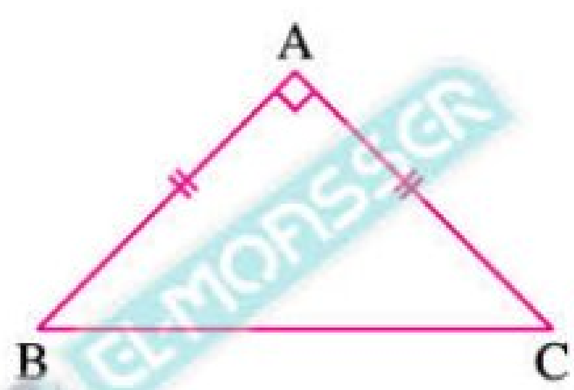
4 The time in which the particle covers a distance of 5 meters from the beginning of the motion.

Model 3

Answer the following questions :

1 **Choose the correct answer from those given :**

1 **In the opposite figure :**



ABC is a right-angled triangle at A and isosceles

, then : tan C = ...

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

2 The perpendicular distance between the two straight lines : $y - 3 = 0$, $y + 2 = 0$ equals .... length units.

(a) 1 (b) 2 (c) 3 (d) 5

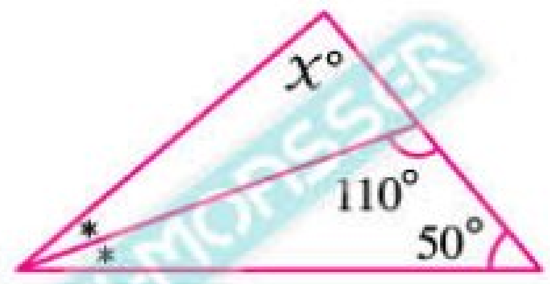
3 Δ ABC in which : m ($\angle$ B) = 5 m ($\angle$ A) = 90° , then m ($\angle$ C) = ...

(a) 72° (b) 50° (c) 90° (d) 45°

4 The slope of the straight line whose equation : $2X - 3y + 5 = 0$ equals ....

(a) $\frac{-3}{2}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

5 **In the opposite figure :**

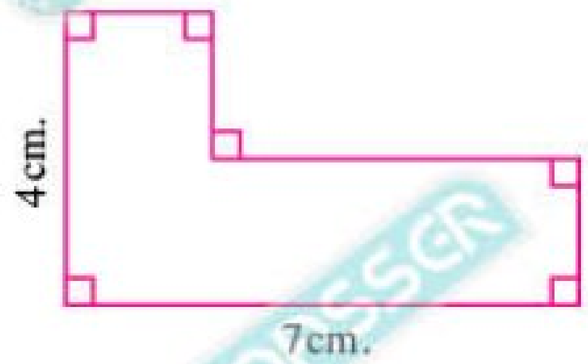


X = ...

(a) 50° (b) 80°

(c) 90° (d) 100°

6 Perimeter of the opposite figure equal ....



(a) 44 cm. (b) 22 cm.

(c) 18 cm. (d) 11 cm.

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2 [a] ABC is a right-angled triangle at B , if $2 AB = \sqrt{3} AC$, then find the main trigonometrical ratios of the angle C.

[b] The straight line whose slope is $\frac{1}{2}$ and intercepts from the positive part of y-axis a part of length two units.

Find : 1 The equation of the straight line.

2 The point of its intersection with the X-axis.

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3 **[a]** Prove that Δ ABC whose vertices are : A (1 , 4) , B (– 1 , – 2) and C (2 , – 3) is right-angled at B , then find its area.

[b] Prove that : $3 \tan^2 45° - 2 \sin 60° \cos 30° = \frac{3}{2}$

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4 [a] **Without using the calculator , find the value of :** $\frac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$

[b] Find the equation of the straight line which passes through the two points (4 , 2) and (– 2 , – 1) , then prove that it passes through the origin point.

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5 **[a]** If the points : A (1 , 0) , B (– 1 , 4) , C (7 , 8) and D (9 , 4) in a perpendicular coordinates plane , prove that figure ABCD is a rectangle and find the length of its diagonal.

[b] If the straight line : $aX + 2y - 3 = 0$ and the straight line which passes through the two points (2 , 3) and (1 , 5) are parallel **, then find the value of :** a

Model 4

Answer the following questions :

1 Choose the correct answer from those given :

1 If m (∠ X) = m (∠ Y) , ∠ X , ∠ Y are complementary , then m (∠ X) = ...

(a) 90°　　(b) 60°　　(c) 45°　　(d) 30°

2 The equation of the straight line which intercepts a part of length 4 units from the positive part of y-axis and is parallel to the straight line : $y = 3X + 5$ is ....

(a) $y = 3X + 4$　　(b) $y = 4X + 3$　　(c) $y = 3X - 4$　　(d) $y = -3X + 4$

3 DEF is a right-angled triangle at E , which of the following relations is false ?

(a) tan D × tan F = 1　　(b) sin D = cos F　　(c) cos D = sin F　　(d) cos D = sin E

4 If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then XA .... XB

(a) >　　(b) <　　(c) =　　(d) ⊥

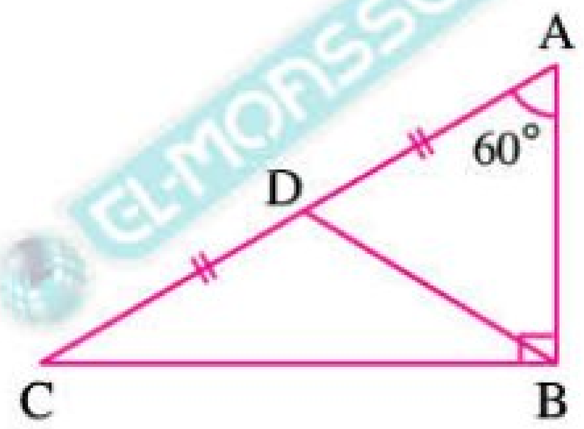
5 If M (1 , 2) is the intersection point of the two diagonals of the parallelogram ABCD where A (2 , 5) , then C = ...

(a) (0 , 2)　　(b) (0 , – 1)　　(c) (– 4 , 1)　　(d) (– 1 , 0)

6 **In the opposite figure :**

If m (∠ ABC) = 90° , m (∠ A) = 60°

and $\overline{BD}$ is a median in Δ ABC , then m (∠ DBC) = ...



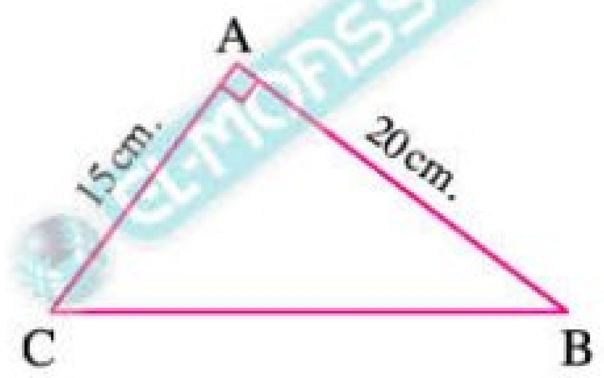
(a) 20°　　(b) 30°

(c) 60°　　(d) 45°

2 [a] In the opposite figure :

ABC is a triangle in which : m (∠ A) = 90°

, AC = 15 cm. and AB = 20 cm.



Prove that : cos C cos B – sin C sin B = zero

[b] Find the equation of the straight line passing through two points A (2 , 3) and B (– 1 , – 3)

Show that for any point C (2 k + 1 , 4 k + 1) , then C ∈ $\overleftrightarrow{AB}$

3 [a] Prove that : The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located on a circle whose centre is M (– 1 , 2) , then find the circumference of the circle where $\pi = 3.14$

[b] Without using the calculator , find : (cos 30° – cos 60°) (sin 30° + sin 60°)

4 **[a]** Find the equation of the straight line passing through the midpoint of $\overline{AB}$ where A (4 , 8) and B (– 2 , 4) and parallel to the straight line whose equation is $2y = 4x - 5$

[b] If $\tan x = \frac{1}{\sqrt{3}}$, x is the measure of an acute angle , **find :** $\sin x \tan\left(\frac{3x}{2}\right) + \cos 2x$

5 **[a]** ABCD is a quadrilateral , X (2 , 3) , Y (m , 3) , Z (1 , – 1) and L (– 4 , n) are the midpoints of $\overline{AB}$, $\overline{AD}$, $\overline{BC}$ and $\overline{DC}$ respectively. **Find :** The value of : m + n

[b] **Prove that :** The points A (4 , 3) , B (7 , 0) and C (1 , – 2) are the vertices of a triangle and if the point D (1 , 2) , then prove that the figure ABCD is a trapezoid and find the ratio between AD and BC

Model 5

Answer the following questions :

1 **Choose the correct answer from those given :**

1 **In the opposite figure :**

The number of the coloured right-angled triangles needed to cover the rectangle surface completely is ....

(a) 4 (b) 6 (c) 8 (d) 12

2 If ABCD is a rhombus and A (– 1 , 7) , B (– 3 , 1) , then the perimeter of the rhombus ABCD = ... length unit.

(a) 40 (b) $4\sqrt{52}$ (c) $8\sqrt{10}$ (d) $2\sqrt{10}$

3 The circumference of the circle with diameter length 14 cm. is .... cm. (where $\pi = \frac{22}{7}$)

(a) 7 (b) 22 (c) 44 (d) 14

4 If ABCD is a parallelogram , then AB + CD = ...

(a) 2 AC (b) 2 BC (c) 2 BD (d) 2 CD

5 In Δ ABC , if m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 5 , then cos B = ...

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

6 If the straight line L is perpendicular to the straight line which passes through the two points (– 1 , 2) and (0 , 5) , then the slope of the straight line L = ...

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

2 **[a] Prove that :**

The points A (1 , 1) , B (2 , 3) and C (0 , – 1) are collinear.

[b] If 3 tan E – 4 $\sin^2 30° = 8 \cos^2 60°$ Find E where E is the measure of an acute angle.

3 **[a]** If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3) , find the coordinates of the point B

[b] In the Cartesian coordinates plane if A (1 , 5) , B ($\mathcal{X}$ – 1 , 3) , C (4 , 7) and D (2 , 1) are four points satisfying $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, **find the value of :** $\mathcal{X}$

4 **[a]** ABC is a right-angled triangle at B in which : BC = 4 cm. and AC = 5 cm.

Deduce that : $\sin^2 A - \cos^2 A = 2 \sin^2 A - 1$

[b] Find the equation of the straight line which intercepts from the positive parts of the coordinate axes «$\mathcal{X}$-axis and y-axis» two parts of lengths 4 and 9 length unit respectively.

5 **[a]** ABC is a triangle whose vertices are A (0 , 6) , B (5 , – 1) and C (– 2 , 1)

Find the equation of the straight line passing through the vertex A and perpendicular to $\overline{BC}$

[b] ABCD is a trapezoid in which : $\overline{AD} \parallel \overline{BC}$, m (∠ B) = 90° , AB = 3 cm.

, AD = 6 cm. and BC = 10 cm.

Prove that : $\cos (\angle DCB) - \tan (\angle ACB) = \frac{1}{2}$

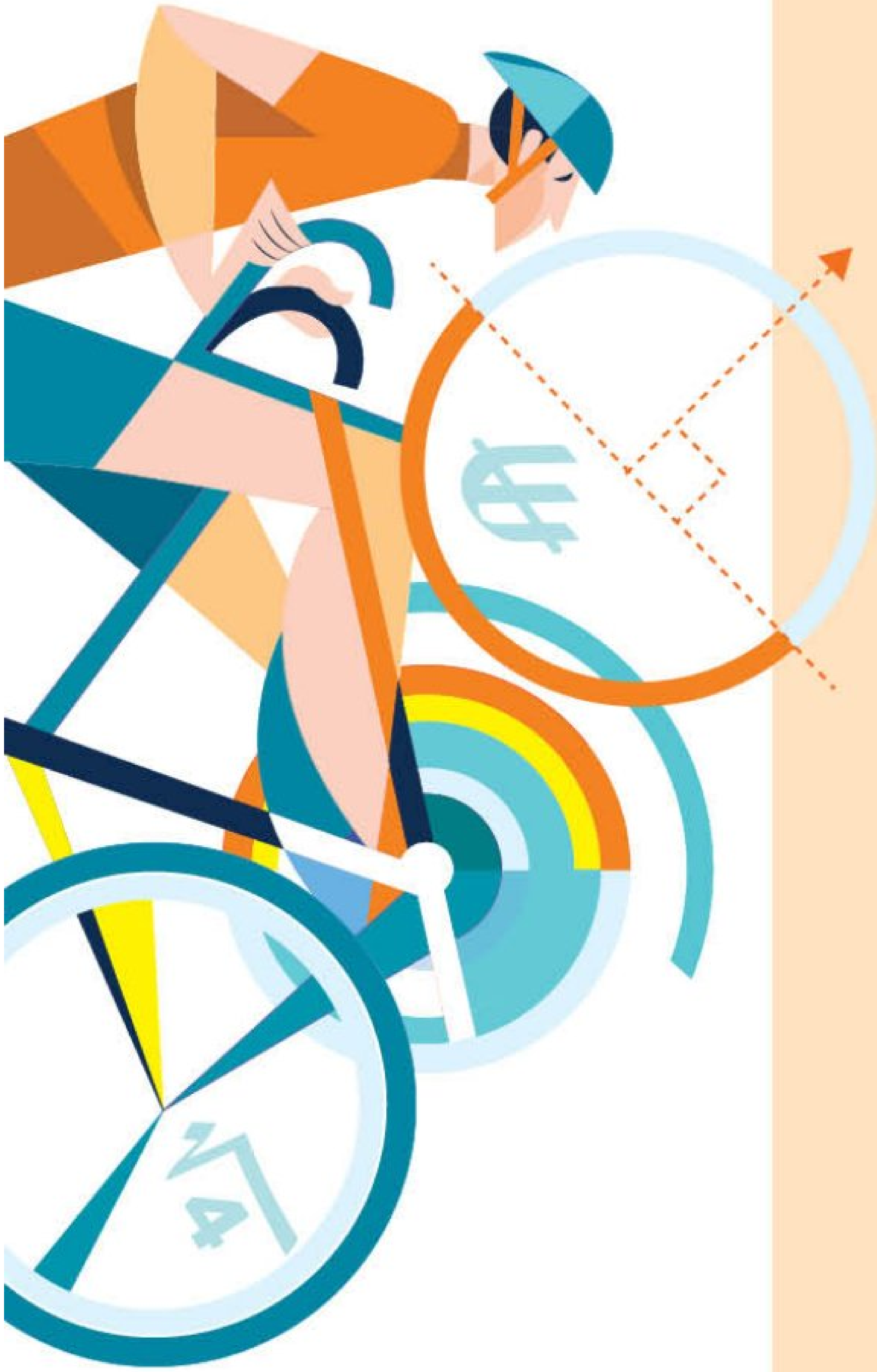
Maths

EL-MOASSER

2023

Answers of

Model Exams



3rd PREP.

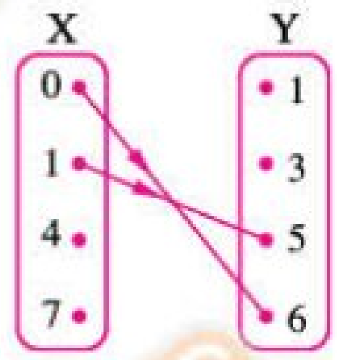
FIRST TERM

Model 1

1

1 b 2 a 3 b 4 d 5 d 6 a

2



[a] $R = \{(0 , 6) , (1 , 5)\}$

, R is not a function because each of

$4 \in X , 7 \in X$ has no image in y

[b] $\because f(x) = 2x - 1$ $\therefore f(2) = 2(2) - 1 = 3$

$, f(1) = 2(1) - 1 = 1$ $\therefore 3f(1) = 3$

$\therefore f(2) - 3f(1) = 3 - 3 = 0$

3

[a] $\because y \propto x$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\therefore \frac{\frac{5}{3}}{\frac{3}{4}} = \frac{\frac{1}{6}}{x_2}$ $\therefore x_2 = \frac{3}{40}$

[b] Let the number be x $\therefore \frac{3 - x}{7 - x} = \frac{7 - x}{19 - x}$

$\therefore (7 - x)^2 = (3 - x)(19 - x)$

$\therefore 49 - 14x + x^2 = 57 - 22x + x^2$

$\therefore 22x - 14x = 57 - 49$ $\therefore 8x = 8$

$\therefore x = 1$

$\therefore$ The number is : 1

4

[a] $\because \frac{x}{y} = \frac{2}{3}$ $\therefore x = 2m , y = 3m$

$\therefore \frac{3x + 2y}{6y - x} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

[b] $\because \frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m$

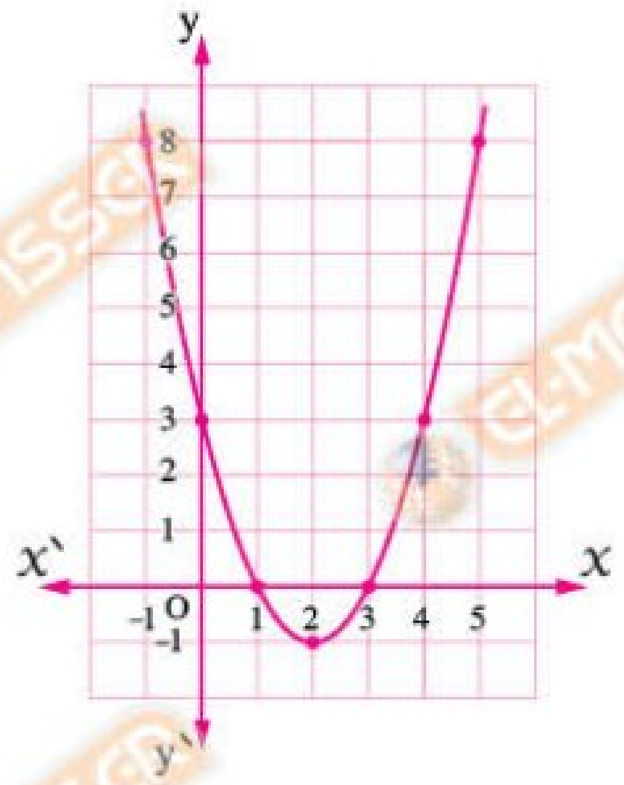
$\therefore a = 4m , b = 5m , c = 3m$

$\therefore \frac{a - b + c}{a + b - c} = \frac{4m - 5m + 3m}{4m + 5m - 3m} = \frac{2m}{6m} = \frac{1}{3}$

5

[a] $f(x) = x^2 - 4x + 3$

x	−1	0	1	2	3	4	5
$f(x)$	8	3	0	−1	0	3	8



From the graph :

1 The minimum value = − 1

2 The equation of the axis of symmetry is : $x = 2$

[b] The mean $(\overline{x}) = \frac{2 + 3 + 6 + 8 + 11}{5} = 6$

x	$x - \overline{x}$	$(x - \overline{x})^2$
2	− 4	16
3	− 3	9
6	0	0
8	2	4
11	5	25
Total		54

The standard deviation $(\sigma) = \sqrt{\frac{54}{5}} \simeq 3.29$

Model 2

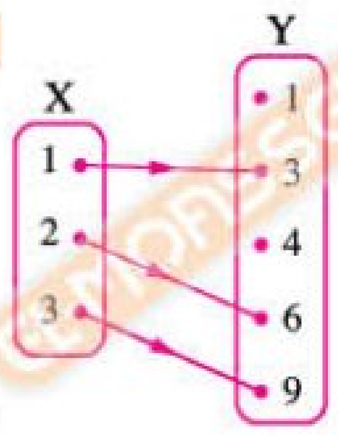
1

1 c 2 c 3 b 4 b 5 c 6 d

2

[a] 1 $R = \{(1 , 3) , (2 , 6) , (3 , 9)\}$

2



3 R is a function because every element in X has only one image in y

[b] Let the number be x $\therefore \frac{5+x^2}{11+x^2}=\frac{3}{5}$

$\therefore 25+5x^2=33+3x^2$

$\therefore 2x^2=8$ $\therefore x^2=4$

$\therefore x=2$ or $x=-2$ (refused)

$\therefore$ The number is 2

3

[a] Subtracting the terms of the 2^{nd} ratio from the 1^{st} ratio.

$\therefore \frac{x+y-y-z}{5-3}=\frac{x-z}{2}=$ one of the given ratios (1)

, and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{5+3+6}=\frac{2x+2y+2z}{14}$

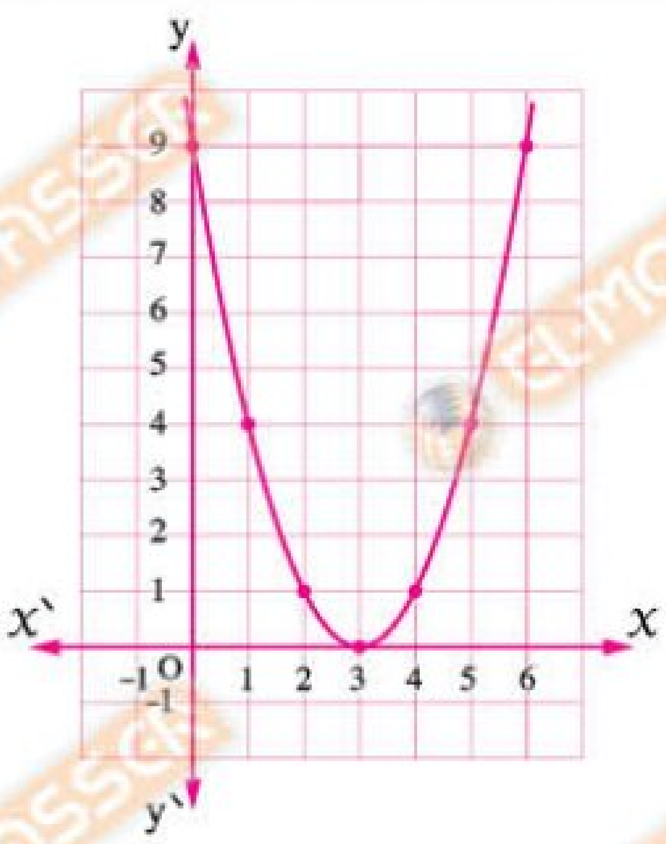
$=\frac{2(x+y+z)}{14}=\frac{x+y+z}{7}$

= one of the ratio (2)

From (1) and (2) : $\therefore \frac{x-z}{2}=\frac{x+y+z}{7}$

[b] $f(x)=(x-3)^2$

x	0	1	2	3	4	5	6
$f(x)$	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : (3 , 0)

2 The minimum value = 0

4

[a] $\because \frac{x}{y}=\frac{y}{z}=m$ $\therefore y=zm$, $x=zm^2$

$\therefore \frac{xz}{y(y+z)}=\frac{zm^2\times z}{zm(zm+z)}=\frac{z^2m^2}{z^2m(m+1)}=\frac{m}{m+1}$ (1)

, $\frac{x}{x+y}=\frac{zm^2}{zm^2+zm}=\frac{zm^2}{zm(m+1)}=\frac{m}{m+1}$ (2)

From (1) and (2) : $\therefore$ The two sides are equals

[b] $\because b \propto \frac{1}{x^2}$ $\therefore b=\frac{m}{x^2}$

$\therefore y=1+\frac{m}{x^2}$

, $\because y=17$ when $x=\frac{1}{2}$ $\therefore 17=1+\frac{m}{\left(\frac{1}{2}\right)^2}$

$\therefore 17=1+4m$ $\therefore 16=4m$

$\therefore m=4$ $\therefore y=1+\frac{4}{x^2}$

, when $x=2$ $\therefore y=1+\frac{4}{2^2}=1+\frac{4}{4}=2$

5

[a] 1 $X=\{1\}$, $Y=\{1 , 2 , 3\}$

2 $Y\times X=\{(1 , 1) , (2 , 1) , (3 , 1)\}$

3 $X^2=\{(1 , 1)\}$

[b]

x	k	$x\times$k
0	9	0
1	15	15
2	17	34
3	25	75
4	20	80
5	14	70
Total	**100**	**274**

The mean $(\overline{X})=\frac{274}{100}=2.74$

x	k	$x\times\overline{x}$	$(x\times\overline{x})^2$	$(x-\overline{x})^2\times$k
0	9	– 2.74	7.5076	67.5684
1	15	– 1.74	3.0276	45.414
2	17	– 0.74	0.5476	9.3092
3	25	0.26	0.0676	1.69
4	20	1.26	1.5876	31.752
5	14	2.26	5.1076	71.5064
Total	**100**			**227.24**

The standard deviation $(\sigma)=\sqrt{\frac{227.24}{100}}\approx 1.51$

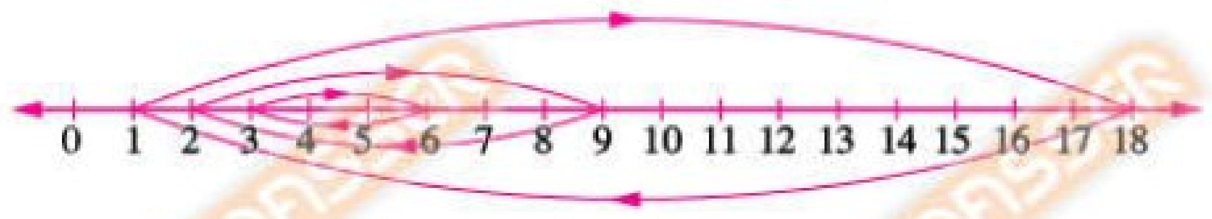
Model 3

1

1 c 2 c 3 a 4 d 5 c 6 b

2

[a] $R = \{(1 , 18) , (18 , 1) , (2 , 9) , (9 , 2) , (6 , 3) , (3 , 6)\}$



[b] $\because \frac{a}{b} = \frac{b}{c} = m$ $\qquad \therefore b = cm , a = cm^2$

$\therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3c^2m^2}{2c^2m^2 - 3c^2m^4} = \frac{c^2(2 - 3m^2)}{c^2m^2(2 - 3m^2)}$

$= \frac{1}{m^2}$ (1)

$, \frac{c}{a} = \frac{c}{cm^2} = \frac{1}{m^2}$ (2)

From (1) and (2) : $\therefore$ The two sides are equals

3

[a] 1 $\because y \propto \frac{1}{X}$ $\qquad \therefore X y = m$

$, \because y = 2$ when $X = 6$ $\qquad \therefore 2 \times 6 = m$

$\therefore m = 12$ $\qquad \therefore X y = 12$

2 when $y = 8$ $\qquad \therefore 8 X = 12$

$\therefore X = \frac{3}{2}$

[b] $\because f(X) = 9 - X$ $\qquad \therefore f(2) = 9 - 2 = 7$

$, f(3) = 9 - 3 = 6$ $\qquad , f(4) = 9 - 4 = 5$

4

[a] Let the number is : X $\qquad \therefore \frac{1 + X}{7 + X} = \frac{7 + X}{25 + X}$

$\therefore (1 + X)(25 + X) = (7 + X)^2$

$\therefore 25 + 26X + X^2 = 49 + 14X + X^2$

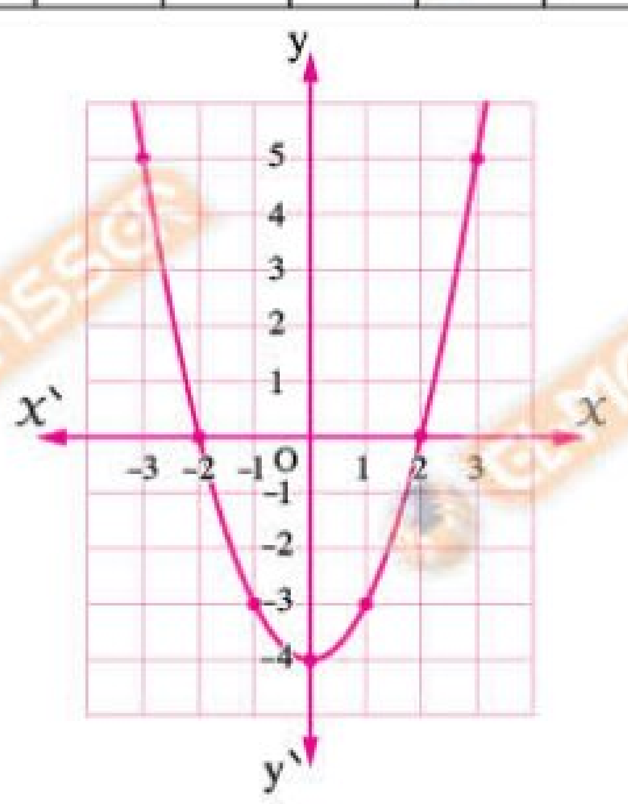
$\therefore 26X - 14X = 49 - 25$

$\therefore 12X = 24$ $\qquad \therefore X = 2$

$\therefore$ The number is : 2

[b] $f(X) = X^2 - 4$

f	−3	−2	−1	0	1	2	3
$f(X)$	5	0	−3	−4	−3	0	5



From the graph :

1 The equation of symmetry axis is : $X = 0$

2 The minimum value $= -4$

5

[a] $\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$

multiplying the terms of the 1st ratio by (2) and the terms of the 2nd ratio by (−1) and the terms of the 3rd ratio by (5) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a - b + 5c}{4 - 3 + 20} =$ one of the given ratios.

$\therefore \frac{2a - b + 5c}{21} = \frac{2a - b + 5c}{3X}$

$\therefore 21 = 3X$ $\qquad \therefore X = 7$

[b]

sets	centres of sets (X)	Frequency (k)	$X \times k$
0 –	1	5	5
2 –	3	9	27
4 –	5	15	75
6 –	7	15	105
8 –	9	6	54
Total		**50**	**266**

The mean $(\overline{X}) = \frac{266}{50} = 5.32$

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
1	5	−4.32	18.6624	93.312
3	9	−2.32	5.3824	48.4416
5	15	−0.32	0.1024	1.536
7	15	1.68	2.8224	42.336
9	6	3.68	13.5424	81.2544
Total	50			266.88

The standard deviation $(\sigma) = \sqrt{\frac{266.88}{50}} \approx 2.31$

Model 4

1

1 c 2 b 3 b 4 d 5 c 6 c

2

[a] Let the number be X

$\therefore \frac{3 + X}{5 + X} = \frac{8 + X}{12 + X}$

$\therefore (5 + x)(8 + x) = (3 + x)(12 + x)$

$\therefore 40 + 13x + x^2 = 36 + 15x + x^2$

$\therefore 40 - 36 = 15x - 13x \quad \therefore 4 = 2x$

$\therefore x = 2$

$\therefore$ The number is : 2

[b] $\because \frac{21x - y}{7x - z} = \frac{y}{z}$ $\quad \therefore 21xz - zy = 7xy - zy$

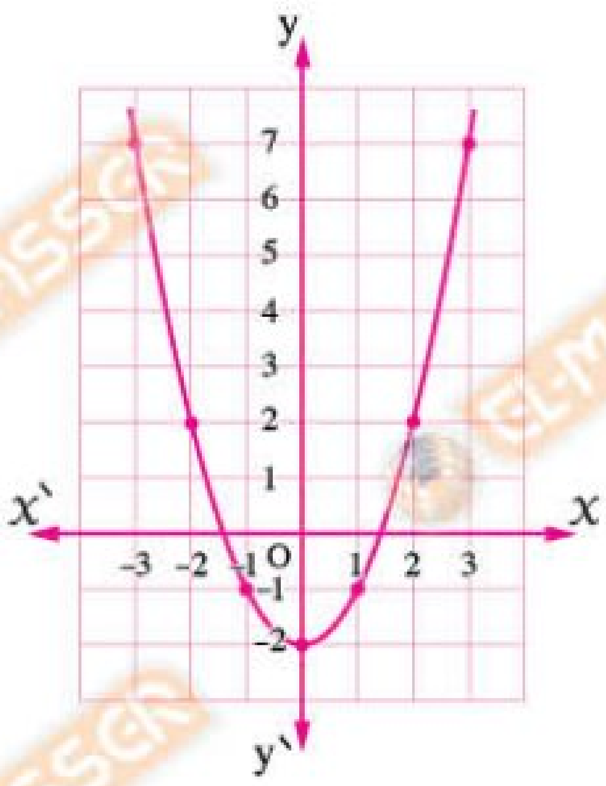
$\therefore 21xz = 7xy$ $\quad \therefore 3z = y$

$\therefore y \propto z$

3

[a] $f(x) = x^2 - 2$

x	−3	−2	−1	0	1	2	3
$f(x)$	7	2	−1	−2	−1	2	7



From the graph :

1 The vertex of the curve is (0 , −2)

2 The minimum value = −2

3 The equation of the line of symmetry is $x = 0$

[b] 1 $f(\sqrt{2}) + 3g(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

2 $\because f(3) = (3)^2 - 3 \times 3 = 9 - 9 = \text{zero}$

, $g(3) = 3 - 3 = \text{zero}$

$\therefore f(3) = g(3) = \text{zero}$

4

[a] Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

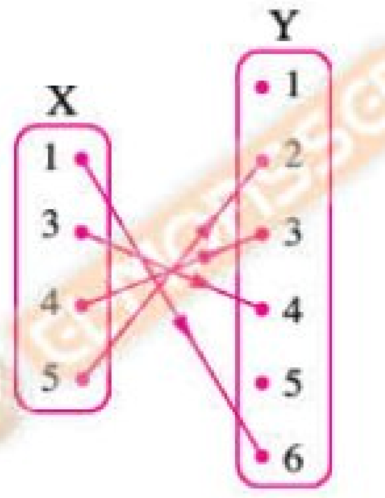
$\therefore c = dm \quad , b = dm^2 , \; a = dm^3$

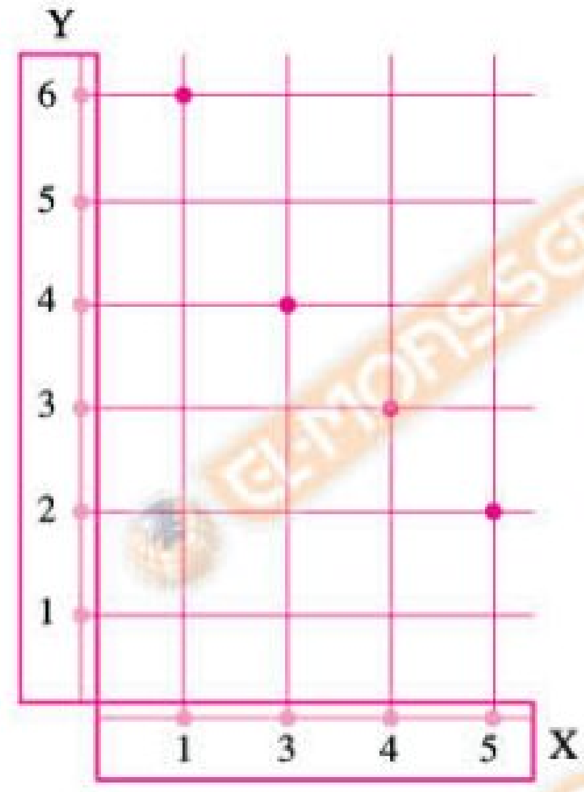
$\therefore \frac{c^2 - d^2}{a - c} = \frac{d^2m^2 - d^2}{dm^3 - dm} = \frac{d^2(m^2 - 1)}{dm(m^2 - 1)} = \frac{d}{m}$ (1)

$, \frac{bd}{a} = \frac{d^2m^2}{dm^3} = \frac{d}{m}$ (2)

From (1) and (2) : $\therefore$ The two sides are equals

[b] $R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$





5

[a] Let $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore x = 3m , y = 4m , z = 5m$

$\therefore \frac{2y - z}{3x - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m} = \frac{3m}{6m} = \frac{1}{2}$

[b]

Age (x)	Number of children (k)	$x \times k$
5	1	5
8	2	16
9	3	27
10	3	30
12	1	12
Total	**10**	**90**

The mean $(\bar{x}) = \frac{90}{10} = 9$ years

x	k	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
5	1	−4	16	16
8	2	−1	1	2
9	3	0	0	0
10	3	1	1	3
12	1	3	9	9
Total	**10**			**30**

The standard deviation $(\sigma) = \sqrt{\frac{30}{10}} \simeq 1.7$ years

Model 5

1

1 b 2 c 3 c 4 d 5 a 6 c

2

[a] $\because f(a) = b \quad \therefore b = a^2 + b \quad \therefore a^2 = 0$

$\therefore a = 0 \quad \therefore a\,b^2 + 5 = 0 \times b^2 + 5 = 5$

[b] Let $\frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm \text{ , } c = dm$

$\therefore \sqrt[3]{\frac{5\,a^3 - 3\,c^3}{5\,b^3 - 3\,d^3}} = \sqrt[3]{\frac{5\,b^3\,m^3 - 3\,d^3\,m^3}{5\,b^3 - 3\,d^3}}$

$= \sqrt[3]{\frac{m^3\,(5\,b^3 - 3\,d^3)}{5\,b^3 - 3\,d^3}} = m \quad (1)$

$, \frac{a + c}{b + d} = \frac{b\,m + d\,m}{b + d} = \frac{m\,(b + d)}{b + d} = m \quad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

3

[a] $R = \{(1 , 3) , (2 , 6) , (3 , 9)\}$

, R is a function from X to Y because each element of X has only one image in Y

and the range $= \{3 , 6 , 9\}$

[b] $\because y = a - 9 \text{ , } y \propto \frac{1}{x^2} \quad \therefore y = \frac{m}{x^2}$

$\therefore \frac{m}{x^2} = a - 9 \quad \therefore m = x^2\,(a - 9)$

$, \because a = 18 \text{ when } x = \frac{2}{3} \quad \therefore m = \frac{4}{9}(18 - 9)$

$\therefore m = \frac{4}{9} \times 9 = 4 \quad \therefore y = \frac{4}{x^2}$

when $x = 1$: $\quad \therefore y = 4$

4

[a] $\because 3\,f(2) + 3\,\ell(x) = 6 \quad \therefore f(2) + \ell(x) = 2$

$\therefore a + 2^2 + c = 2 \quad \therefore a + 4 + c = 2$

$\therefore a + c = -2$

$\therefore 2\,f(0) + 2\,\ell(7) = 2\,[f(0) + \ell(7)] = 2\,[a + (0)^2 + c]$

$= 2\,[a + c] = 2 \times (-2) = -4$

[b] $\because \frac{x}{y} = \frac{2}{3} \quad \therefore x = 2\,m \text{ , } y = 3\,m$

$\therefore \frac{3\,x + 2\,y}{6\,y - x} = \frac{6\,m + 6\,m}{18\,m - 2\,m} = \frac{12\,m}{16\,m} = \frac{3}{4}$

5

[a] Let $\frac{a}{b} = \frac{b}{c} = m \quad \therefore b = c\,m \text{ , } a = c\,m^2$

$\therefore \frac{a^2 + b^2}{b^2 + c^2} = \frac{c^2\,m^4 + c^2\,m^2}{c^2\,m^2 + c^2} = \frac{c^2\,m^2\,(m^2 + 1)}{c^2\,(m^2 + 1)} = m^2 \quad (1)$

$, \frac{a}{c} = \frac{c\,m^2}{c} = m^2 \quad (2)$

From (1) and (2) : $\therefore$ The two sides are equal

[b]

Number of defective units (x)	Number of boxes (k)	$x \times k$
0	3	0
1	16	16
2	17	34
3	25	75
4	20	80
5	19	95
Total	100	**300**

The mean $(\overline{x}) = \frac{300}{100} = 3$ units

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
0	3	-3	9	27
1	16	-2	4	64
2	17	-1	1	17
3	25	0	0	0
4	20	1	1	20
5	19	2	4	76
Total	**100**			**204**

The standard deviation $(\sigma) = \sqrt{\frac{204}{100}} \simeq 1.4$ units

Model 1

1

1 b 2 d 3 c 4 d 5 c 6 b

2

[a] ∵ $AB = \sqrt{(3+3)^2+(4-0)^2} = \sqrt{36+16} = \sqrt{52}$ length unit

, $BC = \sqrt{(1-3)^2+(-6-4)^2} = \sqrt{4+100}$

$= \sqrt{104}$ length unit

, $AC = \sqrt{(1+3)^2+(-6-0)^2} = \sqrt{16+36} = \sqrt{52}$ length unit

∴ AB = AC

∴ Δ ABC is an isosceles triangle

[b] ∵ The slope of the given straight line $= \frac{1}{4}$

∴ The slope of the required straight line = – 4

∴ The equation of the required straight line is :

$y = -4x + c$

, ∵ The straight line passes through the point (1 , 3)

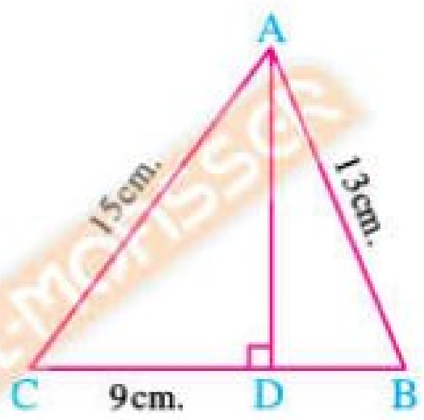
∴ 3 = – 4 (1) + c ∴ c = 7

∴ The equation of the required straight line is :

$y = -4x + 7$

3

[a] ∵ Δ ADC is a right-angled triangle at D



∴ $AD = \sqrt{(15)^2-(9)^2}$

$= \sqrt{225-81}$

= 12 cm.

, From Δ ABD is a right-angled triangle at D

$BD = \sqrt{(13)^2-(12)^2} = \sqrt{169-144} = 5$ cm.

∴ $\frac{\tan(\angle CAD)+\tan(\angle BAD)}{\tan(\angle CAD)-\tan(\angle BAD)} = \frac{\frac{9}{12}+\frac{5}{12}}{\frac{9}{12}-\frac{5}{12}} = \frac{14}{4} = \frac{7}{2}$

[b] 1 ∵ The two diagonals bisect each other

∴ E is the midpoint of $\overline{AC} = \left(\frac{3+1}{2}, \frac{-1+7}{2}\right) = (2 , 3)$

∴ E is the midpoint of $\overline{BD}$

and let D (x , y)

∴ $(2 , 3) = \left(\frac{6+x}{2}, \frac{2+y}{2}\right)$

∴ $\frac{6+x}{2} = 2$ $6 + x = 4$ ∴ $x = -2$

, $\frac{2+y}{2} = 3$ $2 + y = 6$ ∴ y = 4

∴ D = (– 2 , 4)

2 $DE = \sqrt{(-2-2)^2+(4-3)^2} = \sqrt{16+1}$

$= \sqrt{17}$ length unit

4

[a] ∵ $\tan 60° = \sqrt{3}$ (1)

, $2\tan 30° \div (1-\tan^2 30°)$

$= 2 \times \frac{1}{\sqrt{3}} \div \left(1-\left(\frac{1}{\sqrt{3}}\right)^2\right)$

$= \frac{2}{\sqrt{3}} \div \left(1-\frac{1}{3}\right) = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$ (2)

From (1) and (2) : ∴ the two sides are equal.

[b] ∵ The slope of $\overleftrightarrow{XY} = \frac{6+2}{-5-3} = -1$

∴ The slope of the required axis of symmetry = 1

∴ Its equation is : $y = x + c$

, ∵ The axis of symmetry passes through the midpoint of $\overline{XY}$

, The midpoint of $\overline{XY} = \left(\frac{3-5}{2}, \frac{-2+6}{2}\right) = (-1 , 2)$

∴ 2 = – 1 + c ∴ c = 3

∴ The equation of the axis of symmetry of $\overline{XY}$ is : $y = x + 3$

5

[a] ∵ sin E = sin 60° cos 30° – sin 30° cos 60°

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

∴ m (∠ E) = 30°

[b] ∵ The slope of $\overleftrightarrow{AB} = \frac{1-3}{1-4} = \frac{-2}{-3} = \frac{2}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{-3-1}{-5-1} = \frac{-4}{-6} = \frac{2}{3}$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, ∵ B is a common point

∴ A , B and C are collinear.

Model 2

1

1 b 2 b 3 d 4 d 5 a 6 c

2

[a] ∵ The midpoint of $\overline{AB} = \left(\frac{3+1}{2}, \frac{-2-4}{2}\right) = (2 , -3)$

∴ The slope of the required straight line $= \frac{-3-6}{2-1} = -9$

∴ The equation of the required straight line is :

$y = -9x + c$

, ∵ The straight line passes through the point (1 , 6)

∴ 6 = – 9 (1) + c ∴ c = 15

∴ The equation of the required straight line is :

$y = -9x + 15$

[b] ∵ $\sin^3 30° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (1)

, $9\cos^3 60° - \tan^2 45° = 9 \times \left(\frac{1}{2}\right)^3 - (1)^2$

$= \frac{9}{8} - \frac{8}{8} = \frac{1}{8}$ (2)

From (1) and (2) : ∴ The two sides are equal.

3

[a] ∵ Δ ABC is a right-angled triangle at A

∴ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{AC}$ = – 1

∴ $\frac{3+1}{x-3} \times \frac{3+1}{5-3} = -1$ ∴ $\frac{4}{x-3} \times \frac{4}{2} = -1$

∴ $\frac{4}{x-3} = -\frac{1}{2}$ ∴ $3 - x = 8$

∴ $x = -5$

∴ $AB = \sqrt{(-5-3)^2 + (3+1)^2} = \sqrt{64+16}$

$= \sqrt{80}$ length unit.

∴ $AC = \sqrt{(5-3)^2 + (3+1)^2} = \sqrt{4+16} = \sqrt{20}$ length unit.

∴ The area of Δ ABC = $\frac{1}{2}$ AB × AC

$= \frac{1}{2} \times \sqrt{80} \times \sqrt{20}$

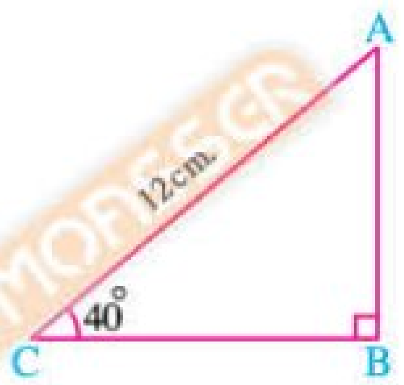
= 20 square units.

[b] 1 ∵ $\sin C = \frac{AB}{AC}$

∴ $\sin 40° = \frac{AB}{12}$

∴ AB = 12 sin 40°

≃ 7.7 cm.



2 ∵ $\cos C = \frac{BC}{AC}$ ∴ $\cos 40° = \frac{BC}{12}$

∴ BC = 12 cos 40° ≃ 9 cm.

4

[a] ∵ $\frac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°}$

$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}} = 1$ (1)

, $\tan^2 45° = (1)^2 = 1$ (2)

From (1) and (2) : ∴ The two sides are equal.

[b] 1 Let A (x , y) , ∵ M is the midpoint of $\overline{AB}$

∴ $(5 , 7) = \left(\frac{x+8}{2} , \frac{y+11}{2}\right)$

∴ $\frac{x+8}{2} = 5$ $x + 8 = 10$ ∴ $x = 2$

, $\frac{y+11}{2} = 7$ $y + 11 = 14$ ∴ $y = 3$

∴ A = (2 , 3)

2 $r = BM = \sqrt{(8-5)^2 + (11-7)^2}$

$= \sqrt{9+16} = 5$ length unit

3 The slope of $\overleftrightarrow{AB} = \frac{11-3}{8-2} = \frac{4}{3}$

∴ The slope of the perpendicular straight line to it = $\frac{-3}{4}$

∴ The equation of required straight line is :

$y = -\frac{3}{4}x + c$

, ∵ The straight line passes through the point B (8 , 11)

∴ $11 = \frac{-3}{4} \times 8 + c$ ∴ c = 17

∴ The equation of required straight line is :

$y = \frac{-3}{4}x + 17$

5

1 1 metre

2 The velocity of the particle = the slope of the straight line passing through

the two points (0 , 1) , $(6 , 3\frac{1}{2}) = \frac{3\frac{1}{2} - 1}{6 - 0} = \frac{5}{12}$ m./sec.

3 $d = \frac{5}{12}t + 1$

4 d = 6 ∴ $6 = \frac{5}{12}t + 1$

∴ $\frac{5}{12}t = 5$ ∴ t = 12 seconds

Model 3

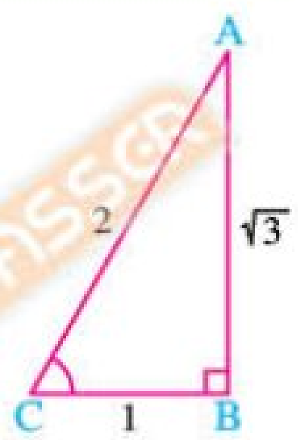
1

1 a 2 d 3 a 4 c 5 c 6 b

2

[a] ∵ $2AB = \sqrt{3}AC$

∴ $\frac{AB}{AC} = \frac{\sqrt{3}}{2}$



Let $AB = \sqrt{3}$ length units

∴ AC = 2 length units , BC = 1 length unit

∴ $\sin c = \frac{\sqrt{3}}{2}$, $\cos c = \frac{1}{2}$, $\tan c = \sqrt{3}$

[b] 1 The straight line is : $y = \frac{1}{2}x + 2$

2 at y = 0 ∴ $0 = \frac{1}{2}x + 2$

∴ $\frac{1}{2}x = -2$ ∴ $x = -4$

∴ The point of its intersection with the x-axis is (– 4 , 0)

3

[a] $\because AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= \sqrt{40}$ length units

, $BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length units

, $AC = \sqrt{(1-2)^2 + (4+3)^2} = \sqrt{1+49}$

$= \sqrt{50}$ length units

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at B

, its area $= \frac{1}{2} \times \sqrt{40} \times \sqrt{10} = 10$ square units

[b] $3 \tan^2 45° - 2 \sin 60° \cos 30°$

$= 3 \times (1)^2 - 2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= 3 - \frac{3}{2} = \frac{3}{2}$

4

[a] $\frac{\cos^2 60° + \cos^2 30 + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°} = \frac{(\frac{1}{2})^2 \times \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}}$

$= \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = \frac{2}{1} = 2$

[b] $\because$ The slope of the straight line $= \frac{2+1}{4+2} = \frac{1}{2}$

$\therefore$ The equation of the straight line is : $y = \frac{1}{2} x + c$

, $\because (4, 2)$ satisfies the equation

$\therefore 2 = \frac{1}{2} \times 4 + c$ $\qquad \therefore c = 0$

$\therefore$ The equation is : $y = \frac{1}{2} x$

, $\because c = 0$

$\therefore$ The length of the intercepted part from y-axis equals zero

$\therefore$ The straight line passes through origin point.

5

[a] $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-0}{-1-1} = -2$

, The slope of $\overleftrightarrow{BC} = \frac{8-4}{7+1} = \frac{1}{2}$

, The slope of $\overleftrightarrow{CD} = \frac{4-8}{9-7} = -2$

, The slope of $\overleftrightarrow{AD} = \frac{4-0}{9-1} = \frac{1}{2}$

$\therefore \overleftrightarrow{AB} // \overleftrightarrow{CD}$, $\overleftrightarrow{BC} // \overleftrightarrow{AD}$

$\therefore$ ABCD is a parallelogram

, $\because$ The slope of $\overleftrightarrow{AB} \times$ The slope of $\overleftrightarrow{BC} = -2 \times \frac{1}{2} = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\qquad \therefore$ ABCD is a rectangle

, $AC = \sqrt{(7-1)^2 + (8-0)^2} = \sqrt{36+64}$

$= 10$ length units

[b] $\because m_1 = \frac{-a}{2}$, $m_2 = \frac{5-3}{1-2} = -2$

, $\because$ two lines are parallel.

$\therefore m_1 = m_2$ $\qquad \therefore \frac{-a}{2} = -2$ $\qquad \therefore a = 4$

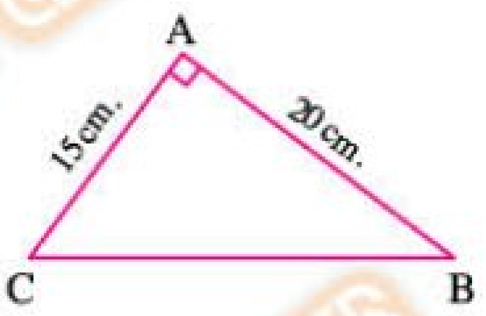
Model 4

1

1 c $\quad$ 2 a $\quad$ 3 d $\quad$ 4 c $\quad$ 5 b $\quad$ 6 b

2

[a] $\because m(\angle A) = 90°$



$\therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm.

$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25}$

$= 0$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{-3-3}{-1-2} = 2$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = 2x + c$

, $\because \overleftrightarrow{AB}$ passes through the point (2 , 3)

$\therefore 3 = 4 + c$ $\qquad \therefore c = -1$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = 2x - 1$

by substituting in the equation of $\overleftrightarrow{AB}$ by $x = 2k + 1$

$\therefore y = 2(2k+1) - 1 = 4k + 2 - 1 = 4k + 1$

$\therefore$ The point C (2 k + 1 , 4 k + 1) satisfies the equation of $\overleftrightarrow{AB}$

$\therefore C \in \overleftrightarrow{AB}$

3

[a] $\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = 5$ length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} = 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = 5$ length units

$\therefore MA = MB = MC$

$\therefore$ A , B and C lie on the circle M whose radius length is 5 length units

$\therefore$ The circumference of the circle $= 2 \pi r$

$= 2 \times 3.14 \times 5$

$= 31.4$ length units

[b] (cos 30° – cos 60°) (sin 30° + sin 60°)

$$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

4

[a] ∵ The slope of the given straight line $= \frac{4}{2} = 2$

∴ The slope of the required straight line = 2

∴ The equation of the required straight line is

$y = 2X + c$

∵ The midpoint of $\overline{AB} = \left(\frac{4-2}{2}, \frac{8+4}{2}\right) = (1, 6)$

∴ (1 , 6) satisfies its equation

∴ $6 = 2 \times 1 + c$ ∴ c = 4

∴ The equation of the required straight line is :

$y = 2X + 4$

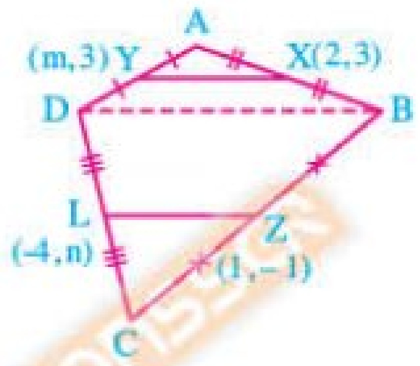
[b] ∵ $\tan X = \frac{1}{\sqrt{3}}$ ∴ $X = 30°$

∴ $\sin X \tan\left(\frac{3X}{2}\right) + \cos(2X)$

$= \sin 30° \tan\left(\frac{3 \times 30°}{2}\right) + \cos(2 \times 30°)$

$= \sin 30° \tan 45° + \cos 60° = \frac{1}{2} \times 1 + \frac{1}{2} = 1$

5



[a] In Δ ABD :

∵ X , Y are the midpoints of $\overline{AB}$, $\overline{AD}$

∴ $\overline{XY} \parallel \overline{BD}$, $XY = \frac{1}{2}$ BD   (1)

, similarly in Δ CBD :

$\overline{ZL} \parallel \overline{BD}$, $ZL = \frac{1}{2}$ BD   (2)

From (1) , (2) : ∴ $\overline{XY} \parallel \overline{ZL}$, XY= ZL

∴ The figure XYLZ is a parallelogram

∴ The midpoint of $\overline{XL}$ is the same midpoint of $\overline{ZY}$

∴ $\left(\frac{2-4}{2}, \frac{3+n}{2}\right) = \left(\frac{m+1}{2}, \frac{3-1}{2}\right)$

∴ $\frac{2-4}{2} = \frac{m+1}{2}$ ∴ m = – 3

, $\frac{3+n}{2} = \frac{3-1}{2}$ ∴ n = –1

∴ m + n = –3 + (– 1) = – 4

[b] ∵ The slope of $\overleftrightarrow{AB} = \frac{0-3}{7-4} = -1$

, the slope of $\overleftrightarrow{BC} = \frac{-2-0}{1-7} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ A , B and C are not collinear

∴ A , B and C are vertices of a triangle

∵ The slope of $\overleftrightarrow{CD} = \frac{2+2}{1-1} = \frac{4}{0}$ (undefined)

, the slope of $\overleftrightarrow{AD} = \frac{3-2}{4-1} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{CD}$

, the slope of $\overleftrightarrow{BC}$ = the slope of $\overleftrightarrow{AD}$

∴ $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

∴ The figure ABCD is a trapezoid

, ∵ $AD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$ length unit

, $BC = \sqrt{(7-1)^2 + (0+2)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ length unit

∴ AD : BC = 1 : 2

Model 5

1

1 d   2 c   3 c   4 d   5 b   6 d

2

[a] ∵ The slope of $\overleftrightarrow{AB} = \frac{3-1}{2-1} = \frac{2}{1} = 2$

, The slope of $\overleftrightarrow{BC} = \frac{-1-3}{0-2} = \frac{-4}{-2} = 2$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, ∵ B is a common point

∴ A , B and C are collinear.

[b] ∵ $3 \tan E - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$

∴ 3 tan E – 1 = 2   ∴ 3 tan E = 3

∴ tan E = 1   ∴ E = 45°

3

[a] Let B (X , y)   ∴ $(6, -4) = \left(\frac{5+X}{2}, \frac{-3+y}{2}\right)$

∴ $\frac{5+X}{2} = 6$ ∴ $5 + X = 12$ ∴ $X = 7$

, $\frac{-3+y}{2} = -4$

∴ – 3 + y = – 8   ∴ y = – 5

∴ B (7 , – 5)

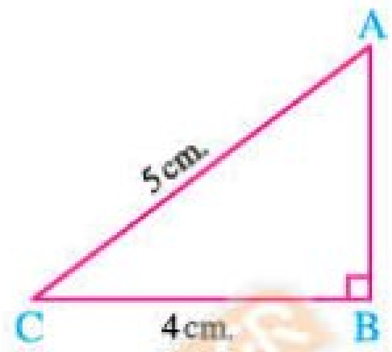
[b] ∵ The slope of $\overleftrightarrow{AD} = \frac{1-5}{2-1} = -4$

, the slope of $\overleftrightarrow{BC} = \frac{7-3}{4-(X-1)} = \frac{4}{5-X}$

∵ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ ∴ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

∴ $-4 = \frac{4}{5-X}$ ∴ $5 - X = -1$ ∴ $X = 6$

4



[a] $\because$ m ($\angle$ B) = 90°

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$\therefore$ AB = 3 cm.

$\therefore \sin^2 A - \cos^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

, $2 \sin^2 A - 1 = 2 \times \left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{7}{25}$

$\therefore \sin^2 A - \cos^2 A = 2 \sin^2 A - 1$

[b] $\because$ The straight line passes through the two points (4 , 0) , (0 , 9)

$\therefore$ The slope of the straight line $= \frac{9-0}{0-4} = -\frac{9}{4}$

and the intercepted part = 9 units from the positive part of y-axis

$\therefore$ The equation of the straight line is :

$y = -\frac{9}{4} x + 9$

5

[a] $\because$ The slope of $\overleftrightarrow{BC} = \frac{1+1}{-2-5} = -\frac{2}{7}$

$\therefore$ The slope of the perpendicular straight line to it $= \frac{7}{2}$

$\therefore$ The equation of the perpendicular to $\overleftrightarrow{BC}$ is

$y = \frac{7}{2} x + c$

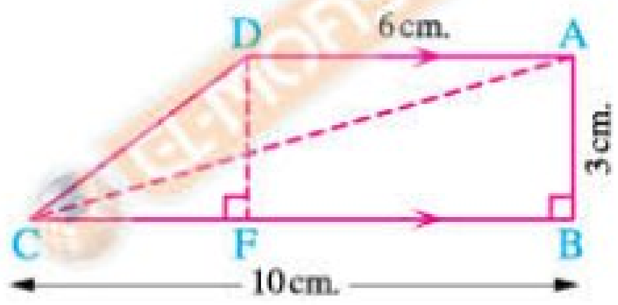
$\because A \in$ the perpendicular to $\overleftrightarrow{BC}$

$\therefore$ (0 , 6) satisfies the equation

$\therefore 6 = \frac{7}{2} \times 0 + c$ $\qquad \therefore c = 6$

$\therefore$ The equation of the perpendicular to $\overleftrightarrow{BC}$ from the point A is $y = \frac{7}{2} x + 6$

[b] Draw $\overline{DF} \perp \overline{BC}$



$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$\therefore$ BF = AD = 6 cm.

$\therefore$ FC = 4 cm. , DF = AB = 3 cm.

$\therefore$ From Δ DFC which is right-angled at F :

$(DC)^2 = 3^2 + 4^2 = 25$

$\therefore$ DC = 5 cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$