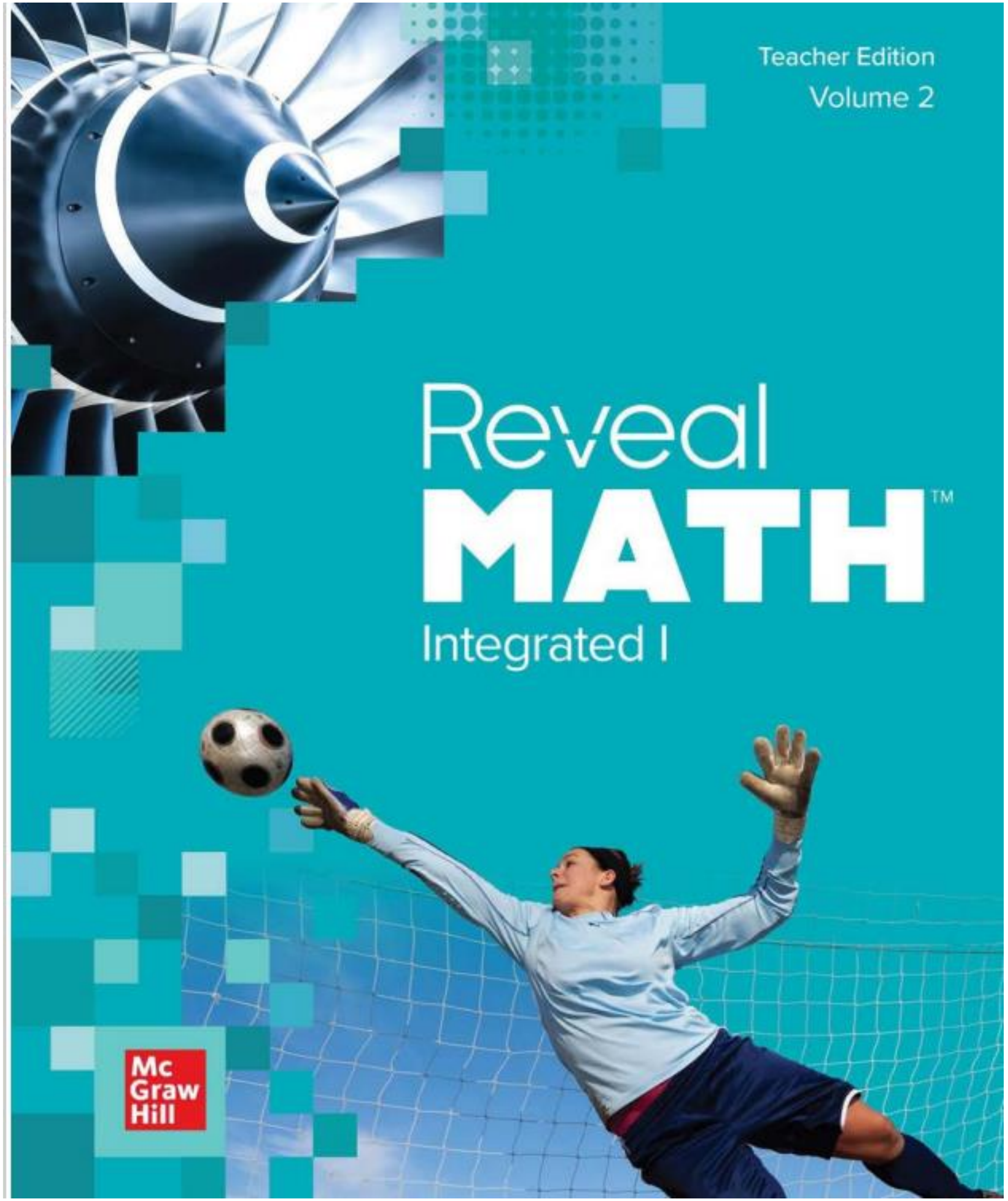


EOT Coverage – 9General

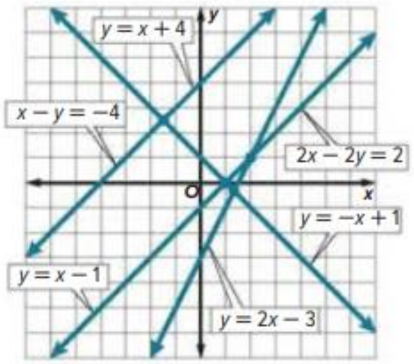
Saif abdulaziz alsabhi
Al Ittihad school

Section	Part	Question** السؤال**	Learning Outcome*** ناتج التعلم***	Reference(s) in the Student Book المرجع في كتاب الطالب: Example/Exercise مثال/تمرين	Page الصفحة
Main Questions - الأسئلة الأساسية	Part 1	1	Determine the number of solutions of a system of linear equations	1 to 10	395
		2	Solve systems of linear inequalities by graphing.	1 to 12	423
		3	Identify points, lines, and planes	1 to 13	565
		4	Identify points, lines, and planes Identify intersections of lines and planes.	20 to 28	566
		5	Calculate measures of line segments.	1 to 9	573
		6	Apply the definition of congruent line segments to find missing values	28 to 33	574
		7	Find the length of a line segment on a number line	1 to 20	581
		8	Analyze figures using the definitions of angles and parts of angles	1 to 4	621
		9	Analyze figures using the characteristics of adjacent angles, linear pairs of angles, and vertical angles	12 to 17	621, 622
		10	Identify and determine characteristics of three-dimensional figures	1 to 6	663
	Part 2	11	Solve systems of equations by eliminating a variable using addition	1 to 21	409
		12	Solve systems of equations by graphing.	11 to 16	395
		13	Solve systems of equations by eliminating a variable using subtraction	1 to 21	409
		14	Find the distance between two points on the coordinate plane	21 to 26	581, 582
		15	Find a point on a directed line segment on a number line that is a given fractional distance from the initial point	1 to 14	589
		16	Find the coordinate of a midpoint on a number line	1 to 16	605
		17	Find missing values using the definition of a segment bisector.	39 to 48	606
		18	Calculate angle measures using the definitions of congruent angles and angle bisectors.	6 to 11	621
		19	Calculate angle measures using the characteristics of complementary and supplementary angles.	1 to 6	631
		20	Find perimeters, circumferences, and areas of two-dimensional geometric shapes	9 to 7	641
	Part 3	21	Solve systems of equations by eliminating a variable using multiplication and addition.	13 to 16	417
		22	Find the coordinates of the midpoint or endpoint of a line segment on the coordinate plane	19 to 38	606
		23	Calculate angle measures using the characteristics of perpendicular lines	7 to 10	631
Bonus Questions - الأسئلة الإضافية		24	A learning outcome from the SoW	Undisclosed	Undisclosed
		25	A learning outcome from the SoW	Undisclosed	Undisclosed

1	Determine the number of solutions of a system of linear equations	1 to 10	395

Examples 1 and 2

Use the graph to determine the number of solutions the system has. Then state whether the system of equations is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



1. $y = x - 1$
$y = -x + 1$

2. $x - y = -4$
$y = x + 4$

3. $y = x + 4$
$2x - 2y = 2$

4. $y = 2x - 3$
$2x - 2y = 2$

Solution

1. $y = x - 1$
   $y = -x + 1$

   SOLUTION:

   Because the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

2. $x - y = -4$
   $y = x + 4$

   SOLUTION:

   Since the graphs of these two lines are the same, there are infinitely many solutions. Therefore, the system is consistent and dependent.

3. $y = x + 4$
   $2x - 2y = 2$

   SOLUTION:

   Because the graphs of these two lines are parallel, there is no solution. Therefore, the system is inconsistent.

4. $y = 2x - 3$
   $2x - 2y = 2$

   SOLUTION:

   Because the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

Examples 3 and 4

Determine the number of solutions the system has. Then state whether the system of equations is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

5. $y = \frac{1}{2}x$
$y = x + 2$

6. $4x - 6y = 12$
$-2x + 3y = -6$

7. $8x - 4y = 16$
$-5x - 5y = 5$

8. $2x + 3y = 10$
$4x + 6y = 12$

9. $y = -\frac{3}{2}x + 5$
$y = -\frac{2}{3}x + 5$

10. $y = x - 3$
$y = -4x + 3$

Solution

5. $y = \frac{1}{2}x$

$y = x + 2$

SOLUTION:

The slope of the first line is $\frac{1}{2}$.

The slope of the second line is 1.

Because the slopes are different, the graphs of these two lines intersect at one point and there is exactly one solution. Therefore, the system is consistent and independent.

6. $4x - 6y = 12$

$-2x + 3y = -6$

SOLUTION:

Write both equations in slope-intercept form.

$4x - 6y = 12$	Original equation	$-2x + 3y = -6$
$-6y = -4x + 12$	Isolate the $y-$term and simplify.	$3y = 2x - 6$
$y = \frac{2}{3}x - 2$	Divide by coefficient of y.	$y = \frac{2}{3}x - 2$

Because the slopes are the same and the y-intercepts are the same, this is the same line. Since the graphs of these two lines are the same, there are infinitely many solutions. Therefore, the system is consistent and dependent.

7. $8x - 4y = 16$

$-5x - 5y = 5$

SOLUTION:

Write both equations in slope-intercept form.

$8x - 4y = 16$	Original equation	$-5x - 5y = 5$
$-4y = -8x + 16$	Isolate the $y-$term and simplify.	$-5y = 5x + 5$
$y = 2x - 4$	Divide by coefficient of y.	$y = -1x - 1$

Because the slopes are different and the y-intercepts are different, the lines intersect. There is exactly one solution. Therefore, the system is consistent and independent.

8. $y = -\frac{3}{2}x + 5$

$y = -\frac{2}{3}x + 5$

SOLUTION:

The slope of the first line is $-\frac{3}{2}$.

The slope of the second line is $-\frac{2}{3}$.

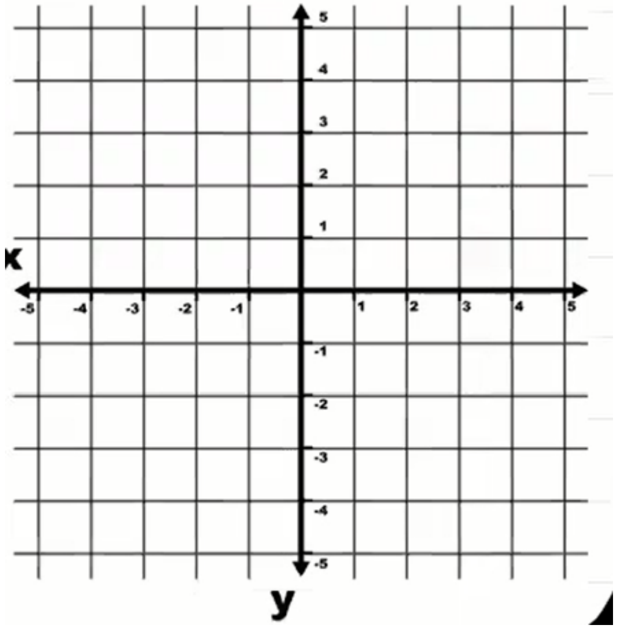
Because the slopes are different, the graphs of these two lines intersect at one point and there is exactly one solution. Therefore, the system is consistent and independent.

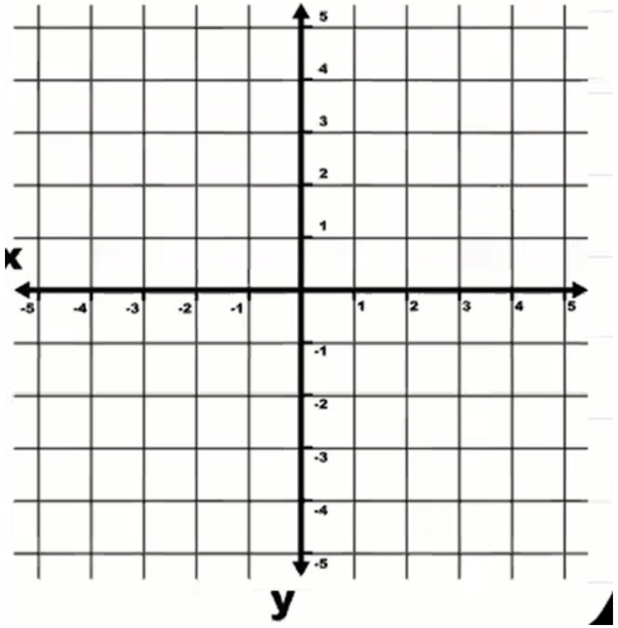
2	Solve systems of linear inequalities by graphing.	1 to 12	423

Examples 1 and 2

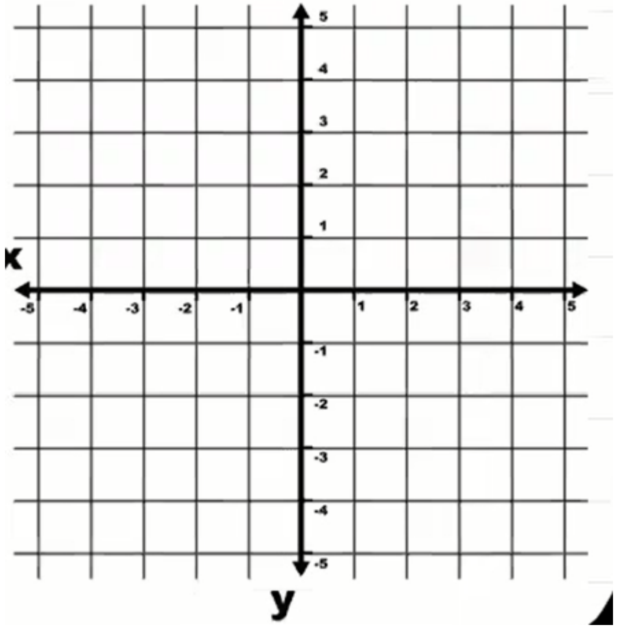
Solve each system of inequalities by graphing.

1. $y < 6$
$y > x + 3$



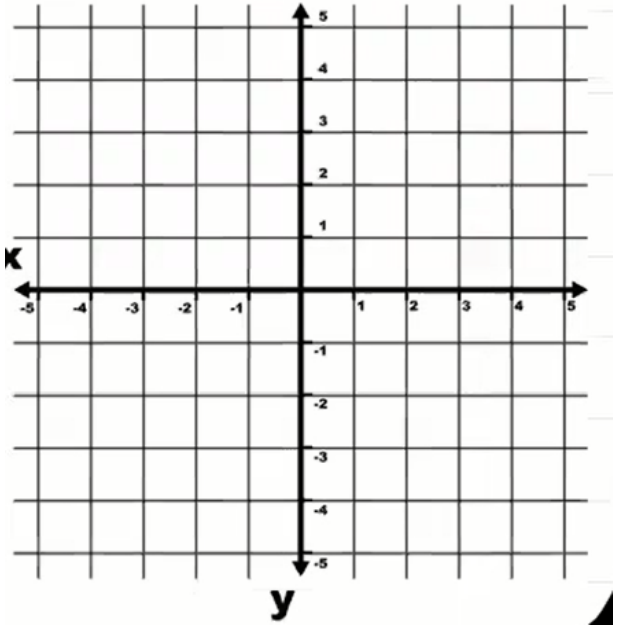


2. $y \geq 0$
$y \leq x - 5$

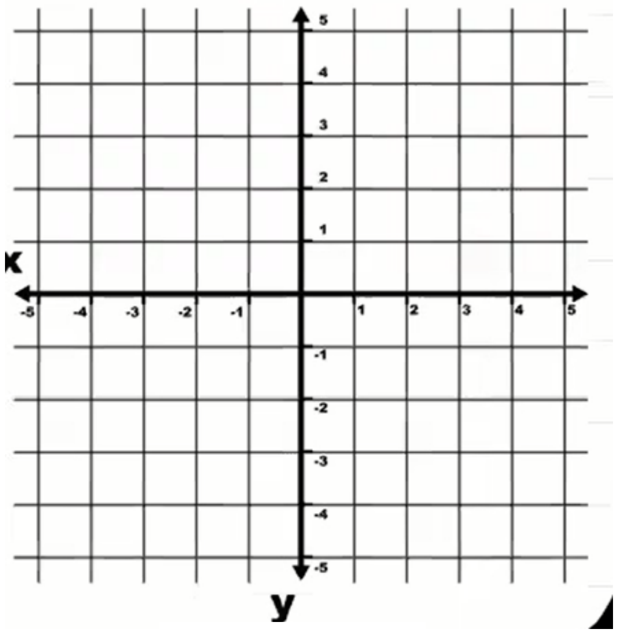


3. $y \leq x + 10$
$y > 6x + 2$

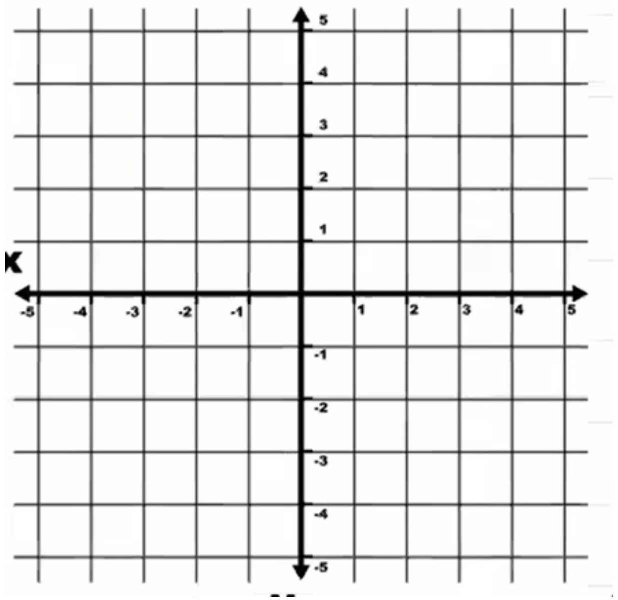
2	Solve systems of linear inequalities by graphing.	1 to 12	423



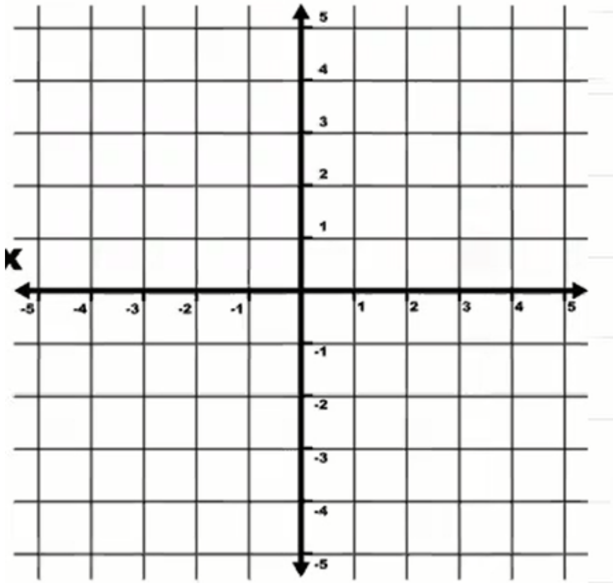
4. $y \geq x + 10$
$y \leq x - 3$



5. $y < 5x - 5$
$y > 5x + 9$

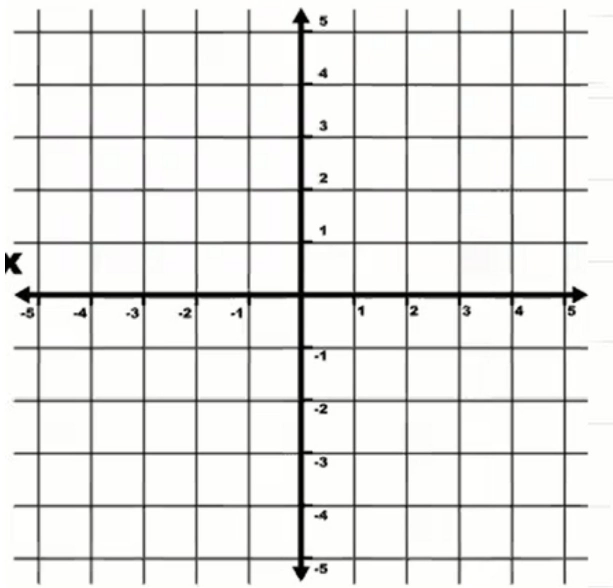


6. $y \geq 3x - 5$
$3x - y > -4$

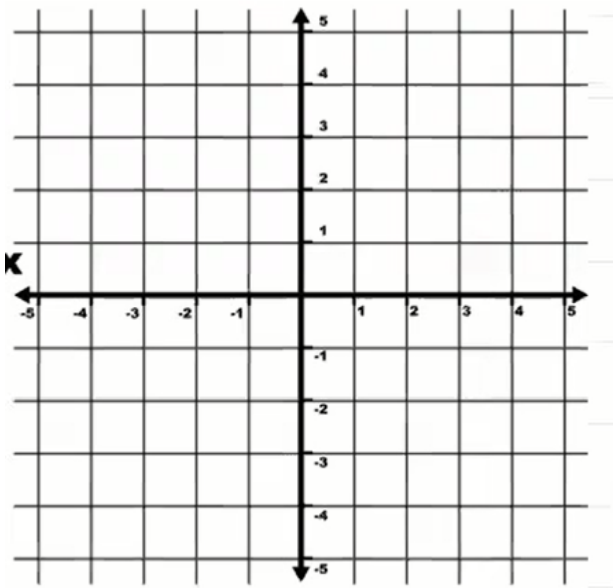


7. $x > -1$
$y \leq -3$

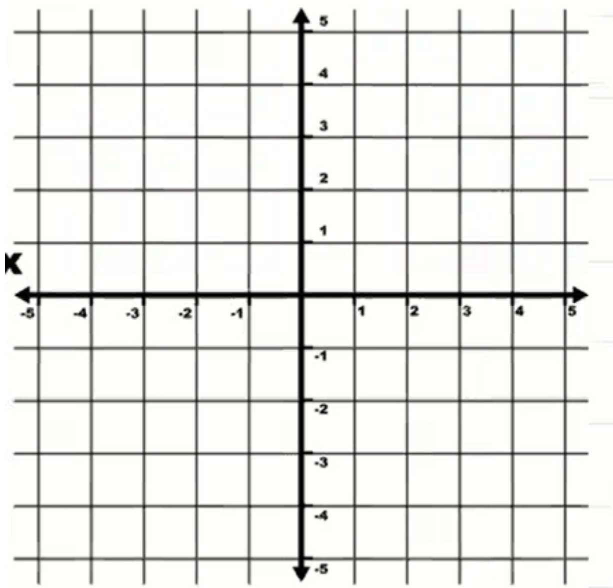
2	Solve systems of linear inequalities by graphing.	1 to 12	423



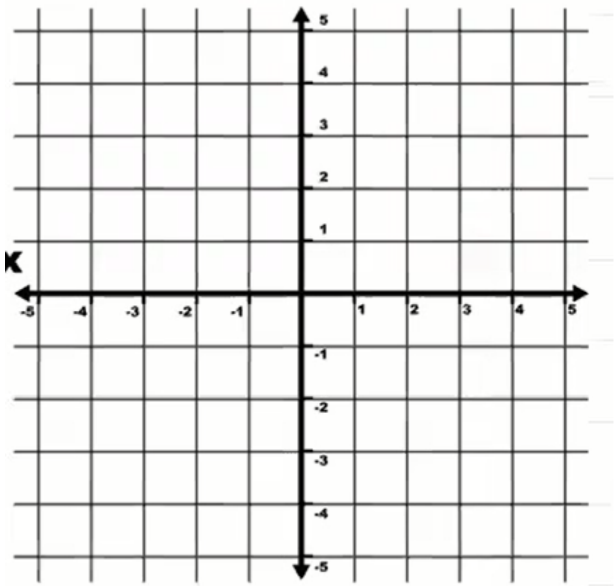
8. $y > 2$
$x < -2$



9. $y > x + 3$
$y \leq -1$



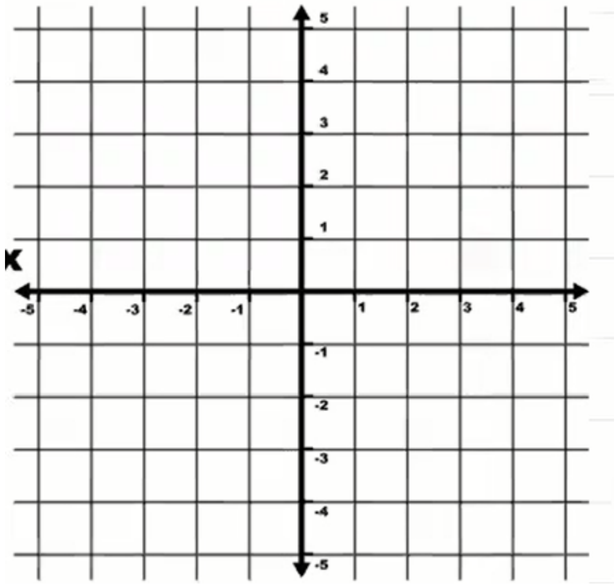
10. $x < 2$
$y - x \leq 2$



11. $x + y \leq -1$
$x + y \geq 3$

2	Solve systems of linear inequalities by graphing.	1 to 12	423

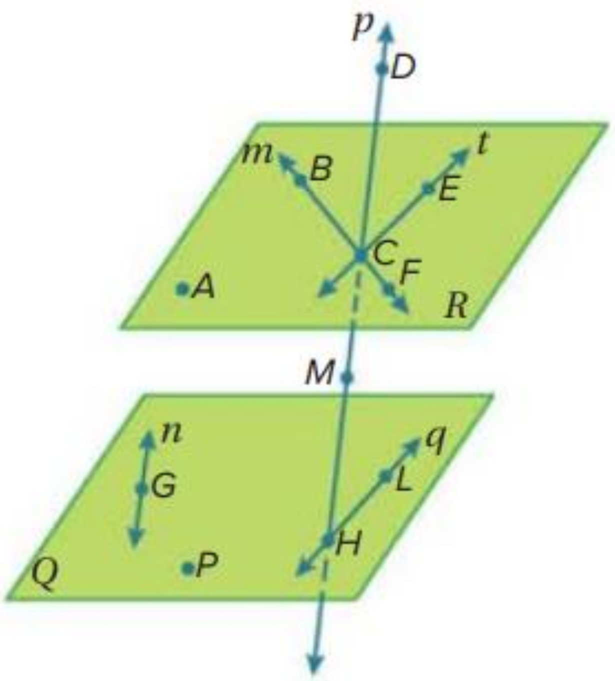
12. $y - x > 4$
$x + y > 2$



3	Identify points, lines, and planes	1 to 13	565

Example 1

Refer to the figure for Exercises 1–7.



1. Name the lines that are only in plane Q.

2. How many planes are labeled in the figure?

3. Name the plane containing the lines m and t.

4. Name the intersection of lines m and t.

5. Name a point that is *not* coplanar with points *A*, *B*, and *C*.

6. Are points *F*, *M*, *G*, and *P* coplanar? Explain.

7. Does line n intersect line q? Explain.

3	Identify points, lines, and planes	1 to 13	565

Example 2

Name the geometric terms modeled by each object or phrase.

8. roof of a house

9. a tabletop

10. bridge support beam

11. a chessboard

12.

13.

4	Identify points, lines, and planes Identify intersections of lines and planes.	20 to 28	566

Example 3

USE TOOLS **Draw and label a figure for each relationship.**

20. Points *X* and *Y* lie on $\overleftrightarrow{CD}$.

21. Two planes do not intersect.

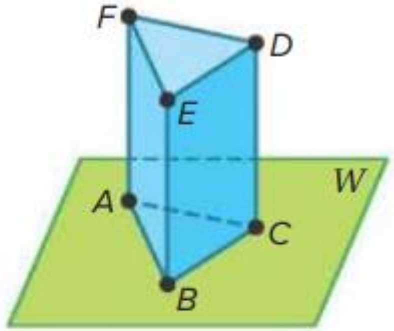
22. Line m intersects plane R at a single point.

23. Three lines intersect at point *J* but do not all lie in the same plane.

24. Points *A*(2, 3), *B*(2, −3), *C*, and *D* are collinear, but *A*, *B*, *C*, *D*, and *F* are not.

Example 4

Refer to the figure for Exercises 25–28.



25. How many planes are shown in the figure?

26. How many of the planes contain points *F* and *E*?

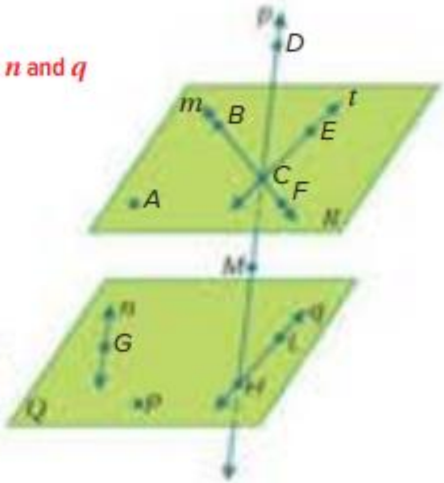
27. Name four points that are coplanar

28. Are points *A*, *B*, and *C* coplanar? Explain.

Solution

Example 1

Refer to the figure for Exercises 1–7.



1. Name the lines that are only in plane Q. Sample answer: n and q
2. How many planes are labeled in the figure? 2
3. Name the plane containing the lines m and t. plane R
4. Name the intersection of lines m and t. point C
5. Name a point that is noncoplanar with points A, B, and C. Sample answer: point P
6. Are points F, M, G, and P coplanar? Explain.
   No; sample answer: Point F lies in plane R, points G and P lie in plane Q, and point M lies between planes R and Q.
7. Does line n intersect line q? Explain.
   Yes; sample answer: Line n intersects line q when the lines are extended.

Example 2

Name the geometric terms modeled by each object or phrase.

8. roof of a house

plane

9. a tabletop

plane

10. bridge support beam

line

11. a chessboard

plane

12.

two planes intersecting in a line

13.

point on a line

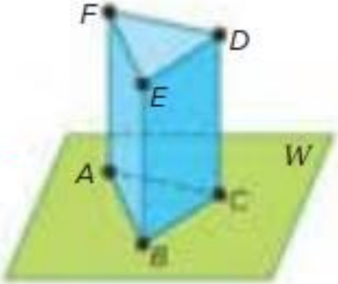
Example 3

USE TOOLS Draw and label a figure for each relationship.

20. Points X and Y lie on $\overleftrightarrow{CD}$. See margin.
21. Two planes do not intersect. See margin.
22. Line m intersects plane R at a single point. See margin.
23. Three lines intersect at point J but do not all lie in the same plane. See margin.
24. Points $A(2, 3)$, $B(2, -3)$, C, and D are collinear, but A, B, C, D, and F are not. See margin.

Example 4

Refer to the figure for Exercises 25–28.



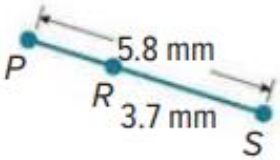
25. How many planes are shown in the figure? 5
26. How many of the planes contain points F and E? 2
27. Name four points that are coplanar. A, B, E, F or B, C, D, E or A, C, D, F
28. Are points A, B, and C coplanar? Explain.
    Yes; sample answer: Points A, B, and C lie in plane W.

5	Calculate measures of line segments.	1 to 9	573

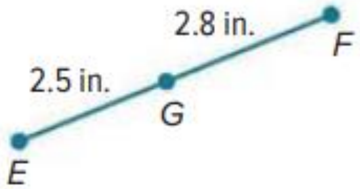
Examples 1 and 2

Find the measure of each segment.

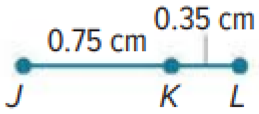
1. $\overline{PR}$



2. $\overline{EF}$

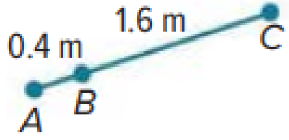


3. $\overline{JL}$

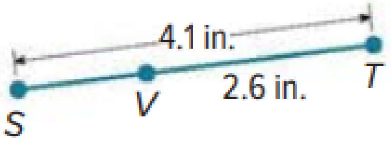


4. $\overline{HJ}$

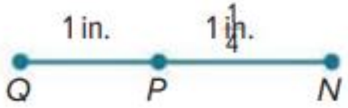
5. $\overline{AC}$



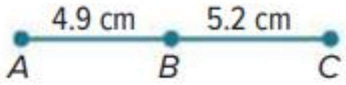
6. $\overline{SV}$



7. $\overline{NQ}$



8. $\overline{AC}$



9. $\overline{GH}$

Solution

Find the measure of each segment.

1. $\overline{PR}$

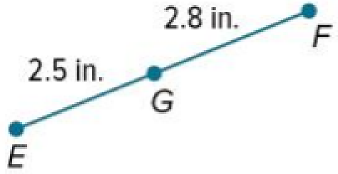
SOLUTION:

$PR + RS = PS$ Betweenness of points
$PR + 3.7 = 5.8$ Substitution
$PR = 2.1$ Subtract 3.7 from each side and simplify.

$PR = 2.1$, so the length of $\overline{PR}$ is 2.1 mm.

ANSWER:
2.1 mm

2. $\overline{EF}$



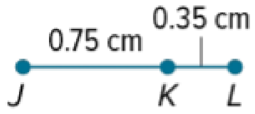
SOLUTION:

$EG + GF = EF$ Betweenness of points
$2.5 + 2.8 = EF$ Substitution
$5.3 = EF$ Add.

$EF = 5.3$, so the length of $\overline{EF}$ is 5.3 in.

ANSWER:
5.3 in.

3. $\overline{JL}$



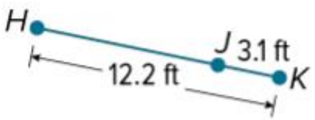
SOLUTION:

$JK + KL = JL$ Betweenness of points
$0.75 + 0.35 = JL$ Substitution
$1.1 = JL$ Addition

$JL = 1.1$, so the length of $\overline{JL}$ is 1.1 cm.

ANSWER:
1.1 cm

4. $\overline{HJ}$



SOLUTION:

$HJ + JK = HK$ Betweenness of points
$HJ + 3.1 = 12.2$ Substitution
$HJ = 9.1$ Subtract 3.1 from each side and simplify.

$HJ = 9.1$, so the length of $\overline{HJ}$ is 9.1 ft.

ANSWER:
9.1ft

5. $\overline{AC}$

SOLUTION:

$AB + BC = AC$ Betweenness of points
$0.4 + 1.6 = AC$ Substitution
$2.0 = AC$ Addition

$AC = 2.0$, so the length of $\overline{AC}$ is 2.0 m.

ANSWER:
2.0 m

6. $\overline{SV}$

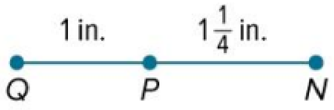
SOLUTION:

$SV + VT = ST$ Betweenness of points
$SV + 2.6 = 4.1$ Substitution
$SV = 1.5$ Subtract 2.6 from each side and simplify.

$SV = 1.5$, so the length of $\overline{SV}$ is 1.5 in.

ANSWER:
1.5 in.

7. $\overline{NQ}$



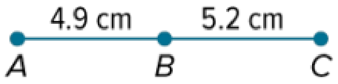
SOLUTION:

$QP + PN = NQ$ Betweenness of points
$1 + 1\frac{1}{4} = NQ$ Substitution
$2\frac{1}{4} = NQ$ Addition

$NQ = 2\frac{1}{4}$, so the length of $\overline{NQ}$ is $2\frac{1}{4}$ in.

ANSWER:
$2\frac{1}{4}$ in.

8. $\overline{AC}$



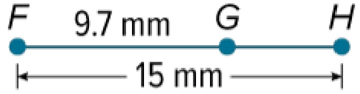
SOLUTION:

$AB + BC = AC$ Betweenness of points
$4.9 + 5.2 = AC$ Substitution
$10.1 = AC$ Addition

$AC = 10.1$, so the length of $\overline{AC}$ is 10.1 cm.

ANSWER:
10.1 cm

9. $\overline{GH}$



SOLUTION:

$FG + GH = FH$ Betweenness of points
$9.7 + GH = 15$ Substitution
$GH = 5.3$ Subtract 9.7 from each side and simplify.

$GH = 5.3$, so the length of $\overline{GH}$ is 5.3 mm.

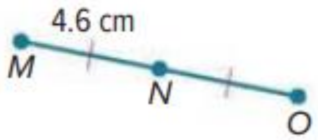
ANSWER:
5.3 mm

6	Apply the definition of congruent line segments to find missing values	28 to 33	574

Example 5

Find the measure of each segment.

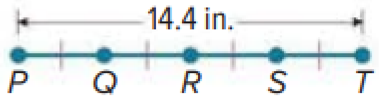
28. $\overline{MO}$



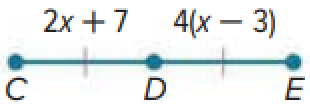
29. $\overline{WY}$

30. $\overline{FG}$

31. $\overline{QT}$

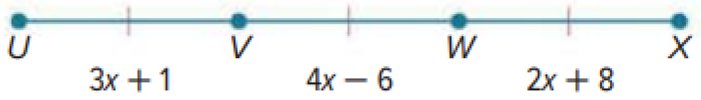


32. $\overline{DE}$



6	Apply the definition of congruent line segments to find missing values	28 to 33	574

33. $\overline{UX}$

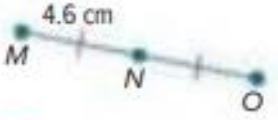


Solution

Example 5

Find the measure of each segment.

28. $\overline{MO}$



9.2 cm

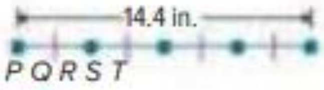
29. $\overline{WY}$

4.4 mm

30. $\overline{FG}$

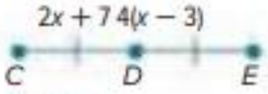
4.2 cm

31. $\overline{QT}$



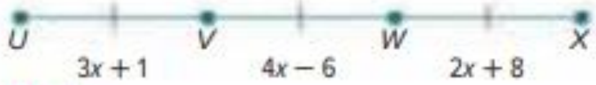
10.8 in.

32. $\overline{DE}$



26 units

33. $\overline{UX}$



66 units

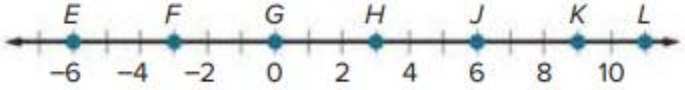
7	Find the length of a line segment on a number line	1 to 20	581

Example 1

Use the number line to find each measure.

1. *JL* **2.** *JK* **3.** *KP* **4.** *NP* **5.** *JP* **6.** *LN*

Use the number line to find each measure.



7. *JK* **8.** *LK* **9.** *FG* **10.** *JG* **11.** *EH* **12.** *LF*

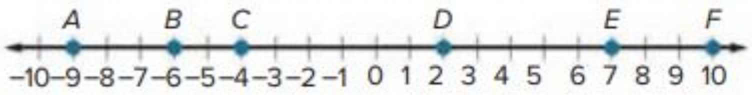
Use the number line to find each measure.

13. *LN* **14.** *JL*

7	Find the length of a line segment on a number line	1 to 20	581

Example 2

Determine whether the given segments are congruent. Write *yes* or *no*.



15. $\overline{AB}$ and $\overline{EF}$

16. $\overline{BD}$ and $\overline{DF}$

17. $\overline{AC}$ and $\overline{CD}$

18. $\overline{AC}$ and $\overline{DE}$

19. $\overline{BE}$ and $\overline{CF}$

20. $\overline{CD}$ and $\overline{DF}$

Practice

Go Online Y ou can complete your homewor

Example 1

Use the number line to find each measure.

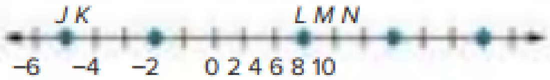
1. *JL* 5
2. *JK* 3
3. *KP* 9
4. *NP* 2
5. *JP* 12
6. *LN* 5

Solution

Use the number line to find each measure.

7. *JK* 3
8. *LK* 2
9. *FG* 3
10. *JG* 6
11. *EH* 9
12. *LF* 14

Use the number line to find each measure.



13. *LN* 6
14. *JL* 8

Example 2

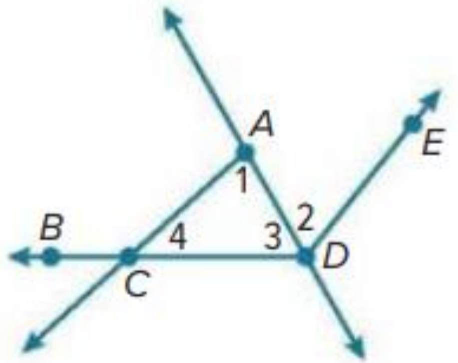
Determine whether the given segments are congruent. Write *yes* or *no*.

15. $\overline{AB}$ and $\overline{EF}$ yes
16. $\overline{BD}$ and $\overline{DF}$ yes
17. $\overline{AC}$ and $\overline{CD}$ no
18. $\overline{AC}$ and $\overline{DE}$ yes
19. $\overline{BE}$ and $\overline{CF}$ no
20. $\overline{CD}$ and $\overline{DF}$ no

8	Analyze figures using the definitions of angles and parts of angles	1 to 4	621

Example 1

Use the figure to identify angles and parts of angles that satisfy each given condition.



1. Name the vertex of ∠1.

2. Name the sides of ∠4.

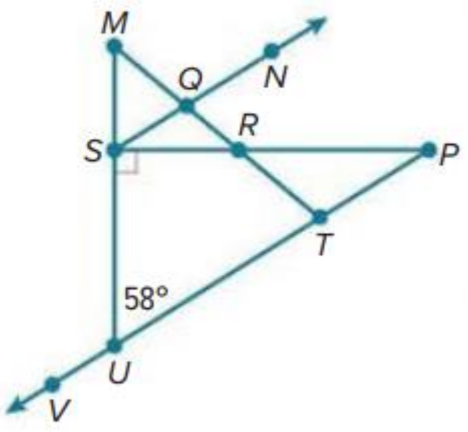
3. What is another name for ∠3?

4. What is another name for ∠*CAD*?

9	Analyze figures using the characteristics of adjacent angles, linear pairs of angles, and vertical angles	12 to 17	621, 622

Example 3

Refer to the figure.



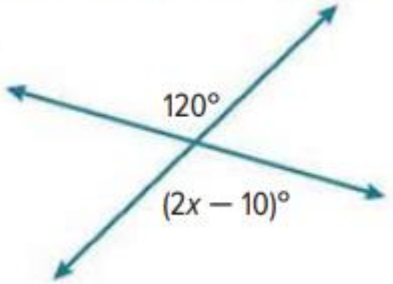
12. Name two adjacent angles.

13. Name two vertical angles.

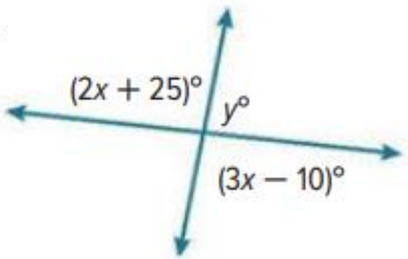
14. Find $m\angle SUV$.

Find the value of each variable.

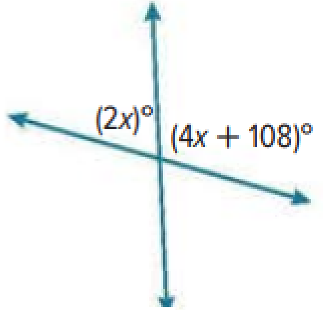
15.



17.



16.



Example 1

Use the figure to identify angles and parts of angles that satisfy each given condition.

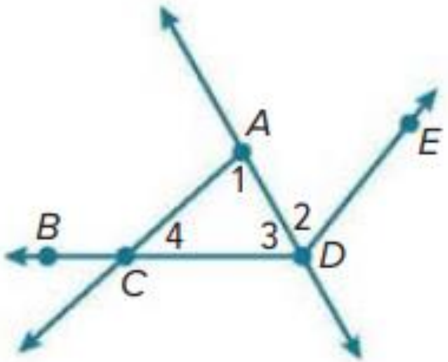
Solution

1. Name the vertex of ∠1. A

2. Name the sides of ∠4. $\overrightarrow{CA}$, $\overrightarrow{CD}$

3. What is another name for ∠3? ∠ADC, ∠CDA

4. What is another name for ∠CAD? ∠1, ∠DAC



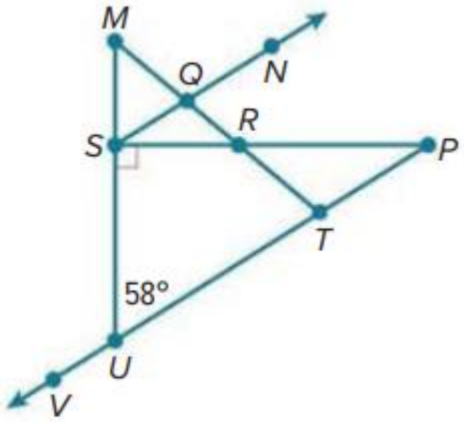
Example 3

Refer to the figure.

12. Name two adjacent angles. Sample answer: ∠MQN and ∠NQR

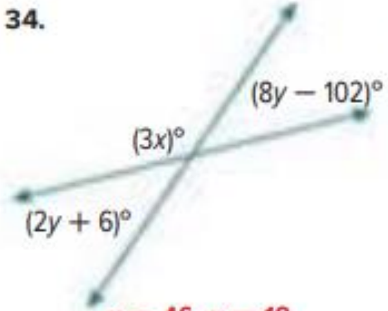
13. Name two vertical angles. Sample answer: ∠SRQ and ∠TRP

14. Find m∠SUV. 122°



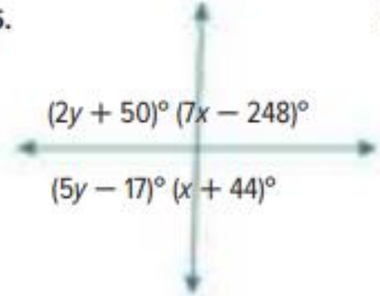
Find the value of each variable.

34.



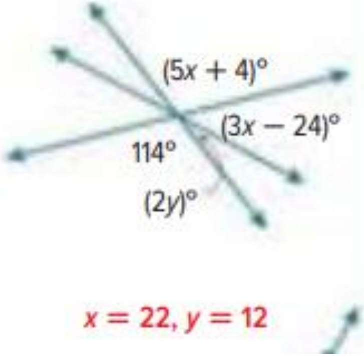
$x = 46$, $y = 18$

35.



$x = 48$, $y = 21$

36.



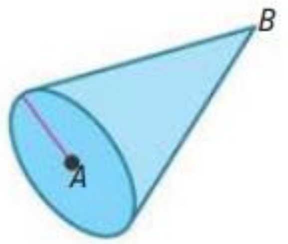
$x = 22$, $y = 12$

10	Identify and determine characteristics of three-dimensional figures	1 to 6	663

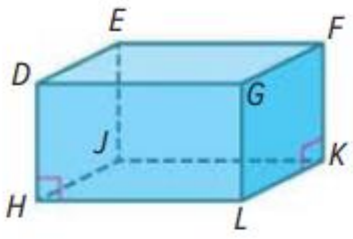
Example 1

Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

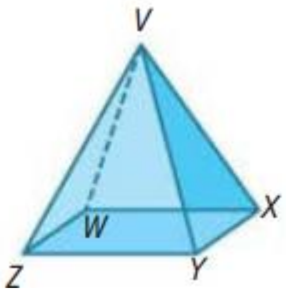
1.



2.



3.



Example 2

Identify the three-dimensional figure that can model each object. State whether the model is or is not a polyhedron.

4.

5.

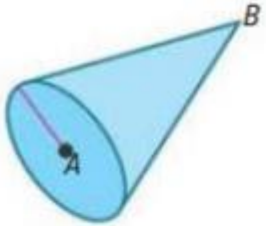
6.

Solution

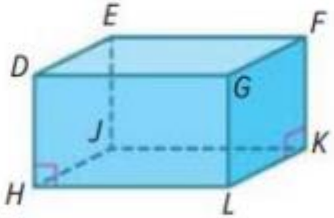
Example 1

Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

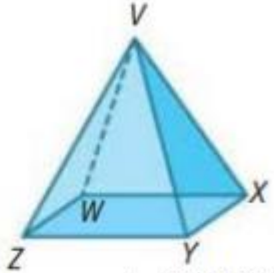
1.



2.



3.



1. *SOLUTION:*

The solid has a curved surface, so it is not a polyhedron. The base is a circle connected by a curved surface to a single vertex, so it is a cone.

ANSWER:

not a polyhedron; cone

The solid is formed by polygonal faces, so it is a polyhedron. The bases are rectangles and the faces a rectangles, so it is a rectangular prism.

bases: rectangles *DEFG* and *HJKL*;

faces:

rectangles *DEFG*, *HJKL*, *DEJH*, *EFKJ*, *FKLG*, *L*

edges: $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GD}$, $\overline{DH}$, $\overline{EJ}$, $\overline{FK}$, $\overline{GL}$, $\overline{HJ}$, $\overline{KL}$, $\overline{LH}$;

vertices: *D*, *E*, *F*, *G*, *H*, *J*, *K*, *L*

SOLUTION:

The solid is formed by polygonal faces, so it is a polyhedron. The base is a square and the faces are triangles that meet at a common vertex, so it is a square pyramid.

base: rectangle *WXYZ*;

faces: rectangle *WXYZ*, ΔVWX, ΔVXY, ΔVYZ, ΔVZW;

edges $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, $\overline{ZW}$, $\overline{WV}$, $\overline{XV}$, $\overline{YV}$, $\overline{ZV}$;

vertices *W*, *X*, *Y*, *Z*, *V*

Solution

Example 2

Identify the three-dimensional figure that can model each object. State whether the model is or is not a polyhedron.

4.

SOLUTION:

The model has a circular base connected by a curved surface to a single vertex, so it is a cone. It is not a polyhedron.

ANSWER:

cone; not a polyhedron

5.

SOLUTION:

The model is a set of all points in space equidistant from its center, so it is a sphere. It is not a polyhedron.

ANSWER:

sphere; not a polyhedron

6.

SOLUTION:

The bases are rectangles and the faces are parallelograms, so the model is a rectangular prism. The solid is formed by polygonal faces, so it is a polyhedron.

ANSWER:

rectangular prism; polyhedron

11	Solve systems of equations by eliminating a variable using addition	1 to 21	409
13	Solve systems of equations by eliminating a variable using subtraction	1 to 21	409

Examples 1, 3

Use elimination to solve each system of equations.

1. $-v + w = 7$
$v + w = 1$

2. $y + z = 4$
$y - z = 8$

3. $-4x + 5y = 17$
$4x + 6y = -6$

4. $5m - 2p = 24$
$3m + 2p = 24$

5. $a + 4b = -4$
$a + 10b = -16$

6. $6r - 6t = 6$
$3r - 6t = 15$

7. $6c - 9d = 111$
$5c - 9d = 103$

8. $11f + 14g = 13$
$11f + 10g = 25$

9. $9x + 6y = 78$
$3x - 6y = -30$

10. $3j + 4k = 23.5$
$8j - 4k = 4$

11. $-3x - 8y = -24$
$3x - 5y = 4.5$

12. $6x - 2y = 1$
$10x - 2y = 5$

11	Solve systems of equations by eliminating a variable using addition	1 to 21	409
13	Solve systems of equations by eliminating a variable using subtraction	1 to 21	409

13. $x - y = 1$
$x + y = 3$

14. $-x + y = 1$
$x + y = 11$

15. $x + 4y = 11$
$x - 6y = 11$

16. $-x + 3y = 6$
$x + 3y = 18$

17. $3x + 4y = 19$
$3x + 6y = 33$

18. $x + 4y = -8$
$x - 4y = -8$

19. $3x + 4y = 2$
$4x - 4y = 12$

20. $3x - y = -1$
$-3x - y = 5$

21. $2x - 3y = 9$
$-5x - 3y = 30$

Solution

Use elimination to solve each system of equations.

1. $-v + w = 7$
$v + w = 1$

SOLUTION:

Since v and $-v$ have opposite coefficients, add the equations to eliminate the variable v.

$-v + w = 7$	Equation 1
$v + w = 1$	Equation 2
$2w = 8$	Add the equations.
$w = 4$	Divide each side by 2.

Solve for the other variable.

$v + w = 1$	Equation 2
$v + 4 = 1$	Substitution
$v = -3$	Subtract 4 from each side.

The solution is $(-3, 4)$.

2. $y + z = 4$
$y - z = 8$

SOLUTION:

Since z and $-z$ have opposite coefficients, add the equations to eliminate the variable z.

$y + z = 4$	Equation 1
$y - z = 8$	Equation 2
$2y = 12$	Add the equations.
$y = 6$	Divide each side by 2.

Solve for the other variable.

$y + z = 4$	Equation 1
$6 + z = 4$	Substitution
$z = -2$	Subtract 6 from each side.

The solution is $(6, -2)$.

3. $-4x + 5y = 17$
$4x + 6y = -6$

SOLUTION:

Since x and $-x$ have opposite coefficients, add the equations to eliminate the variable x.

$-4x + 5y = 17$	Equation 1
$4x + 6y = -6$	Equation 2
$11y = 11$	Add the equations.
$y = 1$	Divide each side by 11.

Solve for the other variable.

$4x + 6y = -6$	Equation 2
$4x + 6(1) = -6$	Substitution
$4x + 6 = -6$	Multiply
$4x = -12$	Subtract 6 from each side.
$x = -3$	Divide each side by 4.

The solution is $(-3, 1)$.

Solution

4. $5m - 2p = 24$
$3m + 2p = 24$

SOLUTION:

Since p and $-p$ have opposite coefficients, add the equations to eliminate the variable p.

$5m - 2p = 24$	Equation 1
$3m + 2p = 24$	Equation 2
$8m = 48$	Add the equations.
$m = 6$	Divide each side by 8.

Solve for the other variable.

$3m + 2p = 24$	Equation 2
$3(6) + 2p = 24$	Substitution
$18 + 2p = 24$	Mutliply
$2p = 6$	Subtract 18 from each side.
$p = 3$	Divide each side by 2.

The solution is (6, 3).

5. $a + 4b = -4$
$a + 10b = -16$

SOLUTION:

Since a and a have the same coefficients, subtract the equations to eliminate the variable a.

$a + 4b = -4$	Equation 1
$a + 10b = -16$	Equation 2
$-6b = 12$	Subtract the equations.
$b = -2$	Divide each side by -6.

Solve for the other variable.

$a + 4b = -4$	Equation 1
$a + 4(-2) = -4$	Substitution
$a - 8 = -4$	Multiply.
$a = 4$	Add 8 to each side.

The solution is (4, −2).

6. $6r - 6t = 6$
$3r - 6t = 15$

SOLUTION:

Since t and t have the same coefficients, subtract the equations to eliminate the variable t.

$6r - 6t = 6$	Equation 1
$3r - 6t = 15$	Equation 2
$3r = -9$	Subtract the equations.
$r = -3$	Divide each side by 3.

Solve for the other variable.

$6r - 6t = 6$	Equation 1
$6(-3) - 6t = 6$	Substitution
$-18 - 6t = 6$	Multiply.
$-6t = 24$	Add 18 to each side.
$t = -4$	Divide each side by -6.

The solution is (−3, −4).

Solution

7. $6c - 9c = 111$
$5c - 9d = 103$

SOLUTION:

Since d and d have the same coefficients, subtract the equations to eliminate the variable d.

$6c - 9d = 111$	Equation 1
$5c - 9d = 103$	Equation 2
$c = 8$	Subtract the equations.

Solve for the other variable.

$6c - 9d = 111$	Equation 1
$6(8) - 9d = 111$	Substitution
$48 - 9d = 111$	Multiply.
$-9d = 63$	Subtract 48 from each side.
$a = -7$	Divide each side by -9.

The solution is $(8, -7)$.

8. $11f + 14g = 13$
$11f + 10g = 25$

SOLUTION:

Since f and f have the same coefficients, subtract the equations to eliminate the variable f.

$11f + 14g = 13$	Equation 1
$11f + 10g = 25$	Equation 2
$4g = -12$	Subtract the equations.
$g = -3$	Divide each side by 4.

Solve for the other variable.

$11f + 14g = 13$	Equation 1
$11f + 14(-3) = 13$	Substitution
$11f - 42 = 13$	Multiply.
$11f = 55$	Add 42 to each side.
$f = 5$	Divide each side by 11.

The solution is $(5, -3)$.

9. $9x + 6y = 78$
$3x - 6y = -30$

SOLUTION:

Since y and $-y$ have opposite coefficients, add the equations to eliminate the variable y.

$9x + 6y = 78$	Equation 1
$3x - 6 = -30$	Equation 2
$12x = 48$	Add the equations.
$x = 4$	Divide each side by 12.

Solve for the other variable.

$3x - 6y = -30$	Equation 2
$3(4) - 6y = -30$	Substitution
$12 - 6y = -30$	Multipy
$-6y = -42$	Subtract 12 from each side.
$y = 7$	Divide each side by -6.

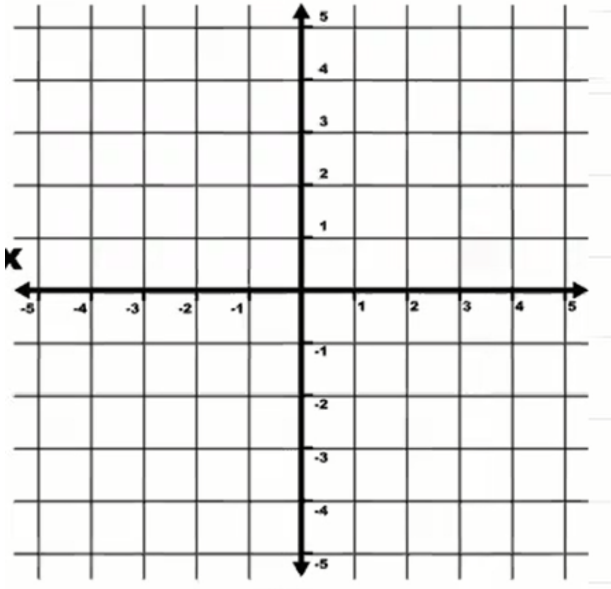
The solution is $(4, 7)$.

12	Solve systems of equations by graphing.	11 to 16	395

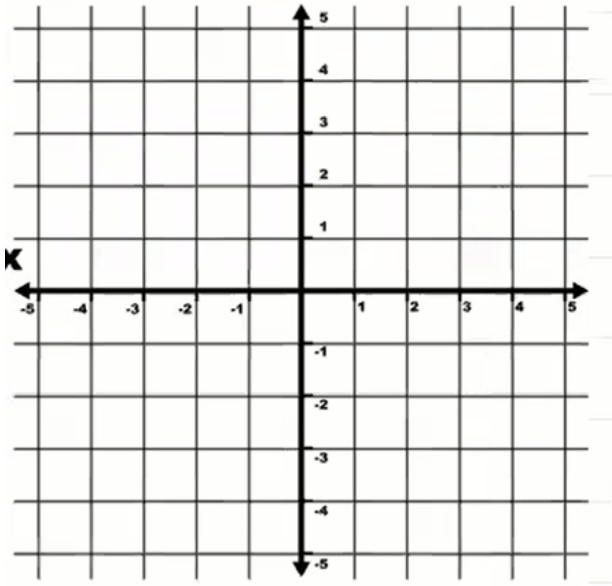
Examples 5 and 6

Graph each system and determine the number of solutions it has. If it has one solution, determine its coordinates.

11. $y = -3$
$y = x - 3$

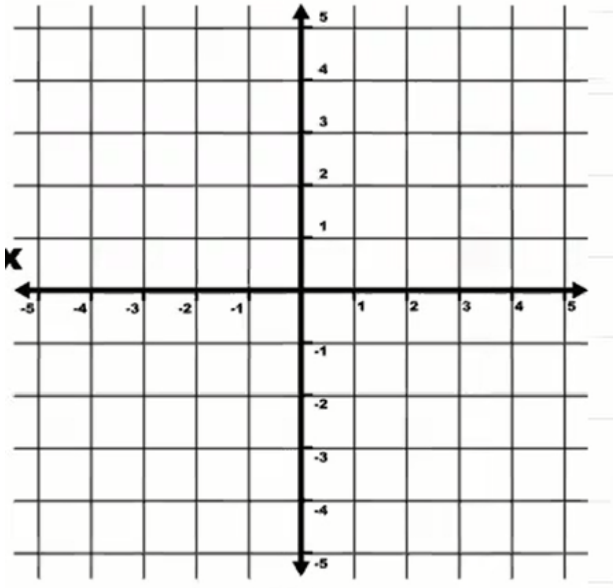


12. $y = 4x + 2$
$y = -2x - 4$

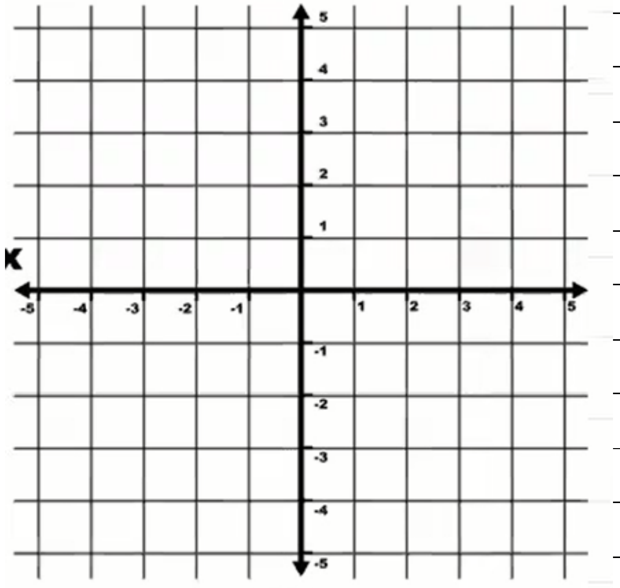


12	Solve systems of equations by graphing.	11 to 16	395

13. $y = x - 6$
$y = x + 2$

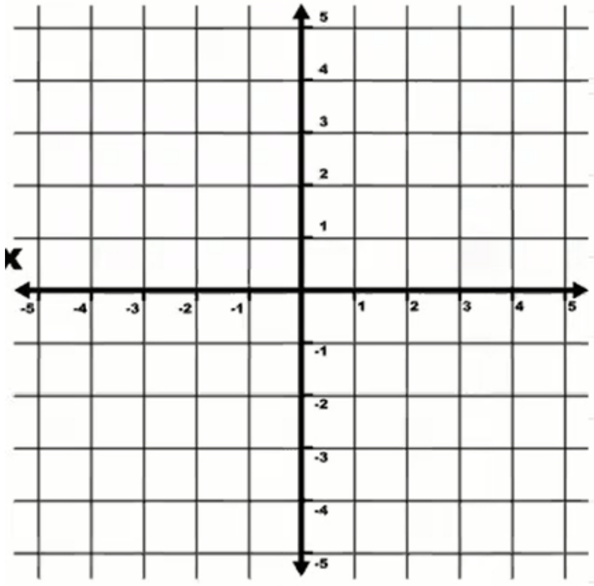


14. $x + y = 4$
$3x + 3y = 12$

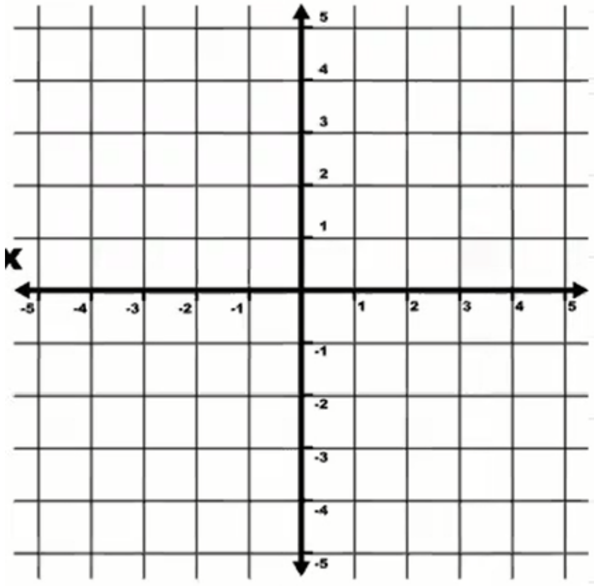


12	Solve systems of equations by graphing.	11 to 16	395

15. $x - y = -2$
$-x + y = 2$



16. $2x + 3y = 12$
$2x - y = 4$

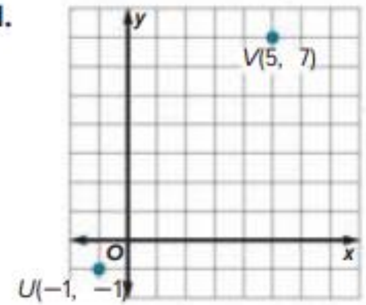


14	Find the distance between two points on the coordinate plane	21 to 26	581, 582

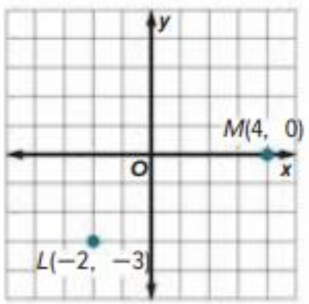
Example 3

Find the distance between each pair of points.

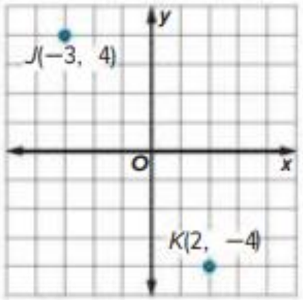
21.



22.



23.



24. $A(2, 6)$, $N(5, 10)$

25. $R(3, 4)$, $T(7, 2)$

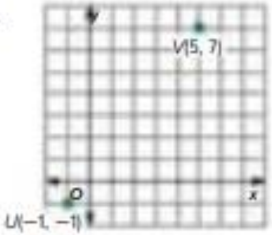
26. $X(-3, 8)$, $Z(-5, 1)$

Solution

Example 3

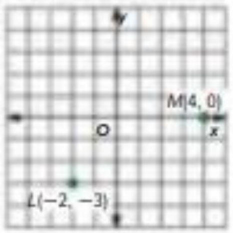
Find the distance between each pair of points.

21.



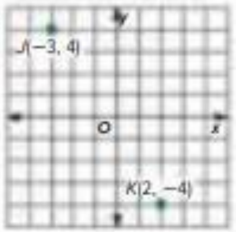
10 units

22.



$\sqrt{45}$ or about 6.7 units

23.



$\sqrt{89}$ or about 9.4 units

24. $A(2, 6)$, $N(5, 10)$
5 units

25. $R(3, 4)$, $T(7, 2)$
$\sqrt{20}$ or about 4.5 units

26. $X(-3, 8)$, $Z(-5, 1)$
$\sqrt{53}$ or about 7.3 units

15	Find a point on a directed line segment on a number line that is a given fractional distance from the initial point	1 to 14	589

Examples 1 and 3

Refer to the number line.

1. Find the coordinate of point *B* that is $\frac{1}{4}$ of the distance from *M* to *J*.

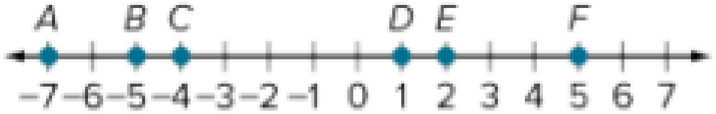
2. Find the coordinate of point *C* that is $\frac{7}{8}$ of the distance from *M* to *J*.

3. Find the coordinate of point *D* that is $\frac{7}{16}$ of the distance from *M* to *J*.

4. Find the coordinate of point *X* such that the ratio of *MX* to *XJ* is 3:1.

5. Find the coordinate of point *X* such that the ratio of *MX* to *XJ* is 2:3.

6. Find the coordinate of point *X* such that the ratio of *MX* to *XJ* is 1:1.



7. Find the coordinate of point *G* that is $\frac{2}{3}$ of the distance from *B* to *D*.

8. Find the coordinate of point *H* that is $\frac{1}{5}$ of the distance from *C* to *F*.

9. Find the coordinate of point *J* that is $\frac{1}{6}$ of the distance from *A* to *E*.

10. Find the coordinate of point *K* that is $\frac{4}{5}$ of the distance from *A* to *F*.

11. Find the coordinate of point *X* such that the ratio of *AX* to *XF* is 1:3.

12. Find the coordinate of point *X* such that the ratio of *BX* to *XF* is 3:2.

Solution

1. Find the coordinate of point B that is $\frac{1}{4}$ of the distance from M to J.

SOLUTION:

Point M is the initial endpoint, and point J is the terminal endpoint.

Use the equation to calculate the coordinate of poin B.

$$\begin{aligned} B &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= 2+\frac{1}{4}(18-2) && x_1=2,\ x_2=18,\text{ and }\frac{a}{b}=\frac{1}{4} \\ &= 6 && \text{Simplify.} \end{aligned}$$

Point B is located at 6 on the number line.

2. Find the coordinate of point C that is $\frac{7}{8}$ of the distance from M to J.

SOLUTION:

Point M is the initial endpoint, and point J is the terminal endpoint.

Use the equation to calculate the coordinate of poin C.

$$\begin{aligned} C &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= 2+\frac{7}{8}(18-2) && x_1=2,\ x_2=18,\text{ and }\frac{a}{b}=\frac{7}{8} \\ &= 16 && \text{Simplify.} \end{aligned}$$

Point C is located at 16 on the number line.

3. Find the coordinate of point D that is $\frac{7}{16}$ of the distance from M to J.

SOLUTION:

Point M is the initial endpoint, and point J is the terminal endpoint.

Use the equation to calculate the coordinate of point D.

$$\begin{aligned} D &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= 2+\frac{7}{16}(18-2) && x_1=2,\ x_2=18,\text{ and }\frac{a}{b}=\frac{7}{8} \\ &= 9 && \text{Simplify.} \end{aligned}$$

Point D is located at 9 on the number line.

7. Find the coordinate of point G that is $\frac{2}{3}$ of the distance from B to D.

SOLUTION:

Point B is the initial endpoint, and point D is the terminal endpoint.

Use the equation to calculate the coordinate of point G.

$$\begin{aligned} G &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= -5+\frac{2}{3}[1-(-5)] && x_1=-5,\ x_2=1,\text{ and }\frac{a}{b}=\frac{2}{3} \\ &= -1 && \text{Simplify.} \end{aligned}$$

Point G is located at –1 on the number line.

8. Find the coordinate of point H that is $\frac{1}{5}$ of the distance from C to F.

SOLUTION:

Point C is the initial endpoint, and point F is the terminal endpoint.

Use the equation to calculate the coordinate of poin H.

$$\begin{aligned} H &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= -4+\frac{1}{5}[5-(-4)] && x_1=-4,\ x_2=5,\text{ and }\frac{a}{b}=\frac{1}{5} \\ &= -2.2 && \text{Simplify.} \end{aligned}$$

Point H is located at –2.2 on the number line.

9. Find the coordinate of point J that is $\frac{1}{6}$ of the distance from A to E.

SOLUTION:

Point A is the initial endpoint, and point E is the terminal endpoint.

Use the equation to calculate the coordinate of point J.

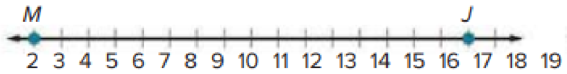
$$\begin{aligned} J &= x_1+\frac{a}{b}(x_2-x_1) && \text{Coordinate equation} \\ &= -7+\frac{1}{6}[2-(-7)] && x_1=-7,\ x_2=2,\text{ and }\frac{a}{b}=\frac{1}{6} \\ &= -5.5 && \text{Simplify.} \end{aligned}$$

Point J is located at –5.5 on the number line.

Solution

Examples 1 and 3

Refer to the number line.



1. Find the coordinate of point B that is $\frac{1}{4}$ of the distance from M to J. 6

2. Find the coordinate of point C that is $\frac{7}{8}$ of the distance from M to J. 16

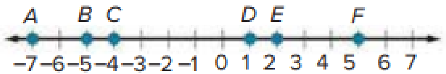
3. Find the coordinate of point D that is $\frac{7}{16}$ of the distance from M to J. 9

4. Find the coordinate of point X such that the ratio of MX to XJ is 3:1. 14

5. Find the coordinate of point X such that the ratio of MX to XJ is 2:3. 8.4

6. Find the coordinate of point X such that the ratio of MX to XJ is 1:1. 10

Refer to the number line.



7. Find the coordinate of point G that is $\frac{2}{3}$ of the distance from B to D. −1

8. Find the coordinate of point H that is $\frac{1}{5}$ of the distance from C to F. −2.2

9. Find the coordinate of point J that is $\frac{1}{6}$ of the distance from A to E. −5.5

10. Find the coordinate of point K that is $\frac{4}{5}$ of the distance from A to F. 2.6

11. Find the coordinate of point X such that the ratio of AX to XF is 1:3. −4

12. Find the coordinate of point X such that the ratio of BX to XF is 3:2. 1

13. Find the coordinate of point X such that the ratio of CX to XE is 1:1. −1

14. Find the coordinate of point X such that the ratio of FX to XD is 5:3. 2.5

16	Find the coordinate of a midpoint on a number line	1 to 16	605

Example 1

Use the number line to find the coordinate of the midpoint of each segment.

1. $\overline{KM}$

2. $\overline{JP}$

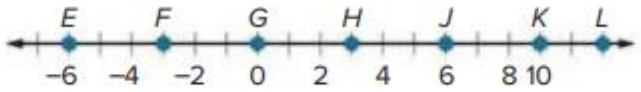
3. $\overline{LN}$

4. $\overline{MP}$

5. $\overline{LP}$

6. $\overline{JN}$

Use the number line to find the coordinate of the midpoint of each segment.



7. $\overline{FK}$

8. $\overline{HK}$

9. $\overline{EF}$

10. $\overline{FG}$

11. $\overline{JL}$

12. $\overline{EL}$

USE TOOLS **Use the number line to find the coordinate of the midpoint of each segment.**

13. $\overline{DE}$

14. $\overline{BC}$

15. $\overline{BD}$

16. $\overline{AD}$

Solution

Example 1

Use the number line to find the coordinate of the midpoint of each segment.

1. $\overline{KM}$ −2

2. $\overline{JP}$ −1

3. $\overline{LN}$ 0.5

4. $\overline{MP}$ 2.5

5. $\overline{LP}$ 1.5

6. $\overline{JN}$ −2

Use the number line to find the coordinate of the midpoint of each segment.

7. $\overline{FK}$ 3

8. $\overline{HK}$ 6

9. $\overline{EF}$ −4.5

10. $\overline{FG}$ −1.5

11. $\overline{JL}$ 8.5

12. $\overline{EL}$ 2.5

USE TOOLS **Use the number line to find the coordinate of the midpoint of each segment.**

13. $\overline{DE}$ 9

14. $\overline{BC}$ 1

15. $\overline{BD}$ 3

16. $\overline{AD}$ $1\frac{1}{2}$

17	Find missing values using the definition of a segment bisector.	39 to 48	606

Examples 5 and 6

Suppose M is the midpoint of $\overline{FG}$ .Find each missing measure.

39. $FM = 5y + 13$, $MG = 5 - 3y$, $FG = ?$

40. $FM = 3x - 4$, $MG = 5x - 26$, $FG = ?$

41. $FM = 8a + 1$, $FG = 42$, $a = ?$

42. $MG = 7x - 15$, $FG = 33$, $x = ?$

43. $FM = 3n + 1$, $MG = 6 - 2n$, $FG = ?$

44. $FM = 12x - 4$, $MG = 5x + 10$, $FG = ?$

45. $FM = 2k - 5$, $FG = 18$, $k = ?$

46. $FG = 14a + 1$, $FM = 14.5$, $a = ?$

17	Find missing values using the definition of a segment bisector.	39 to 48	606

47. $MG = 13x + 1$, $FG = 15$, $x = ?$

48. $FG = 11x - 15.6$, $MG = 10.9$, $x = ?$

Solution

Examples 5 and 6

Suppose M is the midpoint of $\overline{FG}$. Find each missing measure.

39. $FM = 5y + 13$, $MG = 5 - 3y$, $FG = ?$
16

40. $FM = 3x - 4$, $MG = 5x - 26$, $FG = ?$
58

41. $FM = 8a + 1$, $FG = 42$, $a = ?$
2.5

42. $MG = 7x - 15$, $FG = 33$, $x = ?$
4.5

43. $FM = 3n + 1$, $MG = 6 - 2n$, $FG = ?$
8

44. $FM = 12x - 4$, $MG = 5x + 10$, $FG = ?$
40

45. $FM = 2k - 5$, $FG = 18$, $k = ?$
7

46. $FG = 14a + 1$, $FM = 14.5$, $a = ?$
2

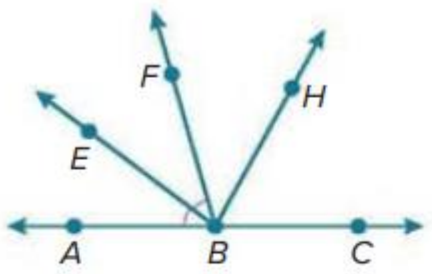
47. $MG = 13x + 1$, $FG = 15$, $x = ?$
0.5

48. $FG = 11x - 15.6$, $MG = 10.9$, $x = ?$
3.4

18	Calculate angle measures using the definitions of congruent angles and angle bisectors.	6 to 11	621

In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$ and $\overrightarrow{BE}$ bisects $\angle ABF$.

6. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.



7. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.

8. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.

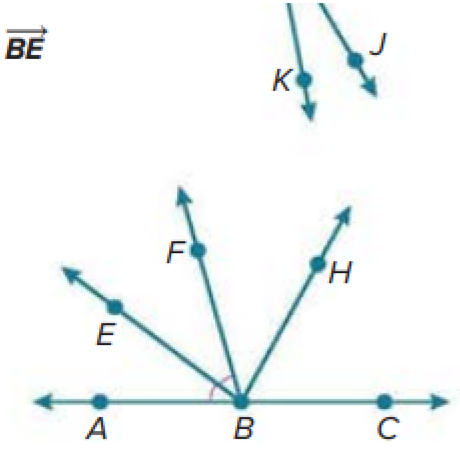
9. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.

10. If $m\angle ABF = 8w - 6$ and $m\angle ABE = 2(w + 11)$, find $m\angle EBF$.

11. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

Solution

In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$ and $\overrightarrow{BE}$ bisects $\angle ABF$.



6. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$. 27°

7. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$. 78°

8. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$. 16°

9. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$. 61°

10. If $m\angle ABF = 8w - 6$ and $m\angle ABE = 2(w + 11)$, find $m\angle EBF$. 47°

11. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$. 56°

19	Calculate angle measures using the characteristics of complementary and supplementary angles.	1 to 6	631

Example 1

1. Find the measures of two supplementary angles if the difference between the measures of the two angles is 35°.

2. $\angle E$ and $\angle F$ are complementary. The measure of $\angle E$ is 54° more than the measure of $\angle F$. Find the measure of each angle.

3. The measure of an angle's supplement is 76° less than the measure of the angle. Find the measures of the angle and its supplement.

4. $\angle Q$ and $\angle R$ are complementary. The measure of $\angle Q$ is 26° less than the measure of $\angle R$. Find the measure of each angle.

5. The measure of the supplement of an angle is three times the measure of the angle. Find the measures of the angle and its supplement.

6. The bascule bridge shown is opening from its horizontal position to its fully vertical position. So far, the bridge has lifted 35° in 21 seconds. At this rate, how much longer will it take for the bridge to reach its vertical position?

Solution

1. Find the measures of two supplementary angles if the difference between the measures of the two angles is 35°.

SOLUTION:

If two angles are supplementary, then the sum of the angle measures is 180°. Let x be the measure of the smaller angle. Then the measure of the larger angle is $180 - x$.

First, solve for x.

$180 - x - x = 35$ Given information

$180 - 2x = 35$ Combine like terms.

$-2x = -145$ Subtract 180 from each side.

$x = 72.5$ Divide each side by -2.

Next, find the measure of the larger angle.

$180 - x = 180 - 72.5$ Substitute 72.5 for x.

$= 107.5$ Solve.

The measures of the angles are 72.5° and 107.5°.

2. $\angle E$ and $\angle F$ are complementary. The measure of $\angle E$ is 54° more than the measure of $\angle F$. Find the measure of each angle.

SOLUTION:

If two angles are complementary, then the sum of the angle measures is 90°. Let $x = \angle F$. Then the measure of $\angle E$ is $x + 54$.

First, solve for x.

$x + x + 54 = 90$ The sum of complementary angles is 90°.

$2x + 54 = 90$ Combine like terms.

$2x = 36$ Subtract 54 from each side.

$x = 18$ Divide each side by 2.

Next, find the measure of $\angle E$.

$x + 54 = 18 + 54$ Substitute 18 for x.

$= 72$ solve.

The measure of $\angle F = 18°$ and $\angle E = 72°$.

3. The measure of an angle's supplement is 76° less than the measure of the angle. Find the measures of the angle and its supplement.

SOLUTION:

If two angles are supplementary, then the sum of the angle measures is 180°. Let x = the measure of the first angle. Then the measure of the supplement is $x - 76$.

First, solve for x.

$x + x - 76 = 180$ Supplementary angle measures sum to 180°.

$2x - 76 = 180$ Combine like terms.

$2x = 256$ Add 76 to each side.

$x = 128$ Divide each side by 2.

Next, find the measure of the supplement.

$x - 76 = 128 - 76$ Substitute 128 for x.

$= 52$ Solve.

The measures of the angles are 128° and 52°.

Solution

4. $\angle Q$ and $\angle R$ are complementary. The measure of $\angle Q$ is 26° less than the measure of $\angle R$. Find the measure of each angle.

SOLUTION:

If two angles are complementary, then the sum of the angle measures is 90°. Let x = the measure of $\angle R$. Then the measure of $\angle Q$ is $x - 26$.

First, solve for x.

$x + x - 26 = 90$	The sum of complementary angles is 90°.
$2x - 26 = 90$	Combine like terms.
$2x = 116$	Add 26 to each side.
$x = 58$	Divide each side by 2.

Next, find the measure of $\angle Q$.

$x - 26 = 58 - 26$	Substitute 58 for x.
$= 32$	Solve.

The measures of $\angle R$ is 58° and $\angle Q$ *is* 32°.

5. The measure of the supplement of an angle is three times the measure of the angle. Find the measures of the angle and its supplement.

SOLUTION:

If two angles are supplementary, then the sum of the angle measures is 180°. Let x be the measure of the smaller angle. Then the supplement is $3x$.

First, solve for x.

$x + 3x = 180$	Supplementary angles sum to 180°.
$4x = 180$	Combine like terms.
$x = 45$	Divide each side by 4.

Next, find the measure of the supplement.

$3x = 3(45)$	Substitute 45 for x.
$= 135$	Solve.

The measures of the angles are 45° and 135°.

6. The bascule bridge shown is opening from its horizontal position to its fully vertical position. So far, the bridge has lifted 35° in 21 seconds. At this rate, how much longer will it take for the bridge to reach its vertical position?

SOLUTION:

The vertical position of the bridge would make an angle of 90°. To find the remaining distance in degrees the bridge has to travel, subtract from 90°.
$90 - 35 = 55$

Set up a proportion and solve.

$\frac{35}{21} = \frac{55}{x}$	Proportional relationship
$35x = 1155$	Cross − multiply.
$x = 33$	Divide each side by 35.

It will take the bridge 33 seconds to reach its vertical position

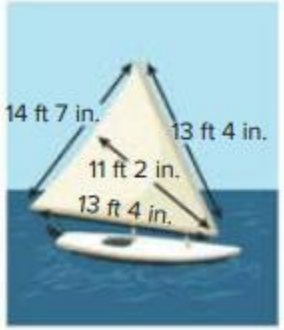
20	Find perimeters, circumferences, and areas of two-dimensional geometric shapes	9 to 7	641

Example 2

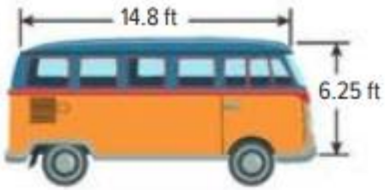
Use a two-dimensional model and the dimensions provided to calculate the perimeter or circumference and area of each object. Round to the nearest tenth, if necessary.

7.

8.



9.



Solution

Example 2

Use a two-dimensional model and the dimensions provided to calculate the perimeter or circumference and area of each object. Round to the nearest tenth, if necessary.

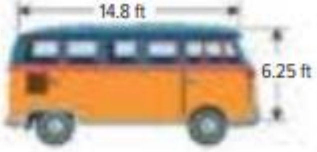
7.

15.7 in.; 19.6 in^2

8.

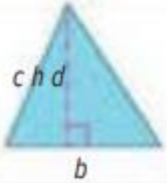
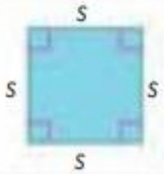
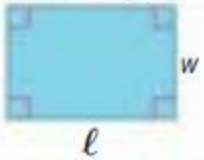
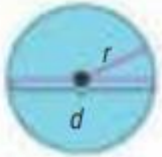
41 ft 3 in.; 81.4 ft^2

9.



42. 1 ft; 92.5 ft^2

Perimeter, Circumference, and Area

Triangle	Square
	
Perimeter $P = b + c + d$ Area $A = \frac{1}{2}bh$	Perimeter $P = s + s + s + s = 4s$ Area $A = s^2$
Rectangle	**Circle**
	
Perimeter $P = \ell + w + \ell + w = 2\ell + 2w$ Area $A = \ell w$	Circumference $C = 2\pi r$ or $C = \pi d$ Area $A = \pi r^2$

21	Solve systems of equations by eliminating a variable using multiplication and addition.	13 to 16	417

Example 3

13. SPORTS The Fan Cost Index (FCI) tracks the average costs for attending sporting events, including tickets, drinks, food, parking, programs, and souvenirs. According to the FCI, a family of four would spend a total of $592.30 to attend two Major League Baseball (MLB) games and one National Basketball Association (NBA) game. The family would spend $691.31 to attend one MLB and two NBA games.

a. Write a system of equations to find the family's costs for each kind of game according to the FCI.

b. Solve the system of equations to find the cost for a family of four to attend each kind of game according to the FCI.

14. ART Mr. Santos, the curator of the children's museum, recently made two purchases of firing clay and polymer clay for a visiting artist to sculpt. Use the table to find the cost of each product per kilogram.

Firing Clay (kg)	Polymer Clay (kg)	Total Cost
5	24	$64.05
25	8	$51.45

a. Write a system of equations to find the cost of each product per kilogram.

b. Solve the system of equations to find the cost of each product per kilogram.

Mixed Exercises

15. Two times a number plus three times another number equals 13. The sum of the two numbers is 7. What are the numbers?

16. Four times a number minus twice another number is −16. The sum of the two numbers is −1. Find the numbers.

22	Find the coordinates of the midpoint or endpoint of a line segment on the coordinate plane	19 to 38	606

Example 3

Find the coordinates of the midpoint of a segment with the given endpoints.

19. (5, 11), (3, 1)

20. (7, −5), (3, 3)

21. (−8, −11), (2, 5)

22. (7, 0), (2, 4)

23. (−5, 1), (2, 6)

24. (−4, −7), (12, −6)

25. (2, 8), (8, 0)

26. (9, −3), (5, 1)

27. (22, 4), (15, 7)

28. (12, 2), (7, 9)

Example 4

Find the coordinates of the missing endpoint if B is the midpoint of $\overline{AC}$.

33. $C(-5, 4)$, $B(-2, 5)$

34. $A(1, 7)$, $B(-3, 1)$

35. $A(-4, 2)$, $B(6, -1)$

36. $C(-6, -2)$, $B(-3, -5)$

37. $A(4, -0.25)$, $B(-4, 6.5)$

38. $C\left(\frac{5}{3}, -6\right)$, $B\left(\frac{8}{3}, 4\right)$

Solution

Example 3

Find the coordinates of the midpoint of a segment with the given endpoints.

19. (5, 11), (3, 1)
(4, 6)

20. (7, −5), (3, 3)
(5, −1)

21. (−8, −11), (2, 5)
(−3, −3)

22. (7, 0), (2, 4)
(4.5, 2)

23. (−5, 1), (2, 6)
(−1.5, 3.5)

24. (−4, −7), (12, −6)
(4, −6.5)

25. (2, 8), (8, 0)
(5, 4)

26. (9, −3), (5, 1)
(7, −1)

27. (22, 4), (15, 7)
(18.5, 5.5)

28. (12, 2), (7, 9)
(9.5, 5.5)

29. (−15, 4), (2, −10)
(−6.5, −3)

30. (−2, 5), (3, −17)
(0.5, −6)

31. (2.4, 14), (6, 6.8)
(4.2, 10.4)

32. (−11.2, −3.4), (−5.6, −7.8)
(−8.4, −5.6)

Example 4

Find the coordinates of the missing endpoint if *B* is the midpoint of $\overline{AC}$.

33. *C*(−5, 4), *B*(−2, 5)
A(1, 6)

34. *A*(1, 7), *B*(−3, 1)
C(−7, −5)

35. *A*(−4, 2), *B*(6, −1)
C(16, −4)

36. *C*(−6, −2), *B*(−3, −5)
A(0, −8)

37. *A*(4, −0.25), *B*(−4, 6.5)
C(−12, 13.25)

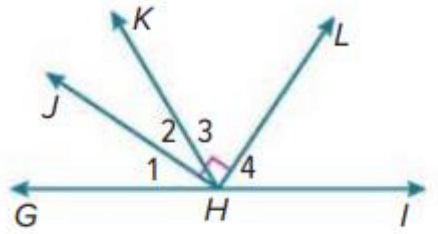
38. $C\left(\frac{5}{3}, -6\right)$, $B\left(\frac{8}{3}, 4\right)$
$A\left(\frac{11}{3}, 14\right)$

23	Calculate angle measures using the characteristics of perpendicular lines	7 to 10	631

Example 2

7. Rays *BA* and *BC* are perpendicular. Point *D* lies in the interior of $\angle ABC$. If $m\angle ABD = (3r + 5)°$ and $m\angle DBC = (5r - 27)°$, find $m\angle ABD$ and $m\angle DBC$.

8. $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point *V*. If $m\angle WVY = (4a + 58)°$ and $m\angle XVY = (2b - 18)°$, find the values of *a* and *b* such that $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$.



9. Refer to the figure at the right. If $m\angle 2 = (a + 15)°$ and $m\angle 3 = (a + 35)°$, find the value of *a* such that $\overrightarrow{HL} \perp \overrightarrow{HJ}$.

10. Rays *DA* and *DC* are perpendicular. Point *B* lies in the interior of $\angle ADC$. If $m\angle ADB = (3a + 10)°$ and $m\angle BDC = 13a°$, find *a*, $m\angle ADB$, and $m\angle BDC$.

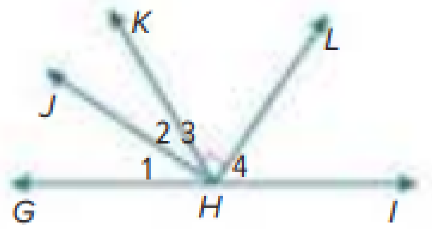
Solution

Example 2

7. Rays *BA* and *BC* are perpendicular. Point *D* lies in the interior of $\angle ABC$. If $m\angle ABD = (3r + 5)^\circ$ and $m\angle DBC = (5r - 27)^\circ$, find $m\angle ABD$ and $m\angle DBC$.
$m\angle ABD = 47^\circ$; $m\angle DBC = 43^\circ$

8. $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point *V*. If $m\angle WVY = (4a + 58)^\circ$ and $m\angle XVY = (2b - 18)^\circ$, find the values of *a* and *b* such that $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$. $a = 8$; $b = 54$

9. Refer to the figure at the right. If $m\angle 2 = (a + 15)^\circ$ and $m\angle 3 = (a + 35)^\circ$, find the value of *a* such that $\overrightarrow{HL} \perp \overrightarrow{HJ}$. $a = 20$



10. Rays *DA* and *DC* are perpendicular. Point *B* lies in the interior of $\angle ADC$. If $m\angle ADB = (3a + 10)^\circ$ and $m\angle BDC = 13a^\circ$, find *a*, $m\angle ADB$, and $m\angle BDC$. $a = 5$; $m\angle ADB = 25^\circ$; $m\angle BDC = 65^\circ$