سلسلة البكالوريا

((التحليل الرياضي))

تمارين تكامل مع الحل

BARA'A BALEK

أ.محمود القسوم

1. $\int \frac{(1-x)^2}{x\sqrt{x}}\,dx$
2. $\int \frac{1}{\sqrt{x.\sqrt{x}}}\,dx$
3. $\int \frac{x^3}{\sqrt[3]{x}}\,dx$
4. $\int \sin^3 x . \sin 2x\,dx$
5. $\int \frac{x}{\sqrt{1+x}}\,dx$
6. $\int \frac{x}{1+x^2}\,dx$
7. $\int \frac{(\ln x)^2}{x}\,dx$
8. $\int \sqrt{\sin x}\,.\cos x\,dx$ $]0,\pi[$
9. $\int \frac{\ln^5 x}{x}\,dx$
10. $\int \tan 2x\,dx$
11. $\int \frac{1}{x^2} . \sin\left(\frac{1}{x}\right) dx$
12. $\int \frac{dx}{\sin^2 3x}$
13. $\int \frac{1}{x^2} . e^{\frac{1}{x}}\,dx$
14. $\int x . \sin 2x\,dx$
15. $\int x^2 . e^x\,dx$
16. $\int x . \ln x\,dx$
17. $\int (x^2+1)\cos x\,dx$
18. $\int 2x . \cos^2 x\,dx$
19. $\int e^{\sqrt{x}}\,dx$
20. $\int \frac{x}{\sqrt{2x+1}}\,dx$
21. $\int \ln x\,dx$
22. $\int \frac{1}{x.\ln x}\,dx$

(23) $\int \frac{1}{\sin 2x} \, dx$ $\left]\frac{\pi}{2}, \pi\right[$

(24) $\int_{0}^{\ln 2} \frac{1}{e^x+1} \, dx$

(25) $\int_{-2}^{-1} \frac{2x-1}{x-1} \, dx$

(26) $\int_{0}^{3} |2x+2| \, dx$

(27) $\int_{0}^{3} |x^2-3x+2| \, dx$

(28) $\int_{-2}^{+2} |x^2+4x| \, dx$

(29) $\int_{0}^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^5 x}$

(30) $\int x . \sqrt{4-x^2} \, dx$

(31) $\int_{0}^{9} \frac{x-1}{1+\sqrt{x}} \, dx$

(32) $\int x . \sin x^2 \, dx$

(33) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x} \, dx$

(34) $\int_{1}^{e^3} \frac{1}{x\sqrt{1+\ln x}} \, dx$

(35) $\int_{1}^{2} \sqrt[3]{(1-x)} \, dx$

(36) $\int \frac{8}{x^3(x-1)} \, dx$

(37) $\int \frac{x^2+x+2}{(x-1)(x^2+1)} \, dx$

(38) $\int 2 . \cos 3x . \sin 2x \, dx$

(39) $\int \cos 2x . \cos x \, dx$

(40) $\int \ln^2 x \, dx$

(41) $\int_{0}^{1} \frac{e^x}{e^x+1} \, dx$

42 $\int x(1+x)^n \, dx$

$n \in N$

43 $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

44 $\int_{-1}^{0} \frac{-2x}{(x-1)^2} \, dx$

45 $\int_0^{\ln 2} \frac{e^x-1}{e^x+1} \, dx$

Baraa Balek

انسخ ..

①

$$\int \frac{(1-x)^2}{x\sqrt{x}}\,dx$$

$$\int (1-x)^2 \cdot x^{-\frac{3}{2}} \cdot dx = \int (1-2x+x^2)\,x^{-\frac{3}{2}}dx$$

$$\int \left(x^{-\frac{3}{2}} - 2x^{1-\frac{3}{2}} + x^{2-\frac{3}{2}}\right)dx$$

$$\int \left(x^{-\frac{3}{2}} - 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right)dx$$

$$= \left[\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} - 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right] + C$$

$$= \left[\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 2\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$-2x^{-\frac{1}{2}} - 4x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$$

$$= \frac{-2}{\sqrt{x}} - 4\sqrt{x} + \frac{2}{3}\sqrt{x^3} + C$$

②

$$\int \frac{1}{\sqrt{x\cdot\sqrt{x}}}\,dx = \int \frac{1}{\sqrt{x\cdot x^{\frac{1}{2}}}}\,dx$$

$$= \int \frac{1}{\sqrt{x^{\frac{3}{2}}}}\,dx = \int \frac{1}{\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}}\,dx$$

$$= \int \frac{1}{x^{\frac{3}{4}}}\,dx = \int x^{-\frac{3}{4}}\,dx$$

$$\frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + C = \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + C$$

$$= 4x^{\frac{1}{4}} + C = 4\sqrt[4]{x} + C$$

③

$$\int \frac{x^3}{\sqrt[3]{x}}\,dx = \int \frac{x^3}{x^{\frac{1}{3}}}\,dx = \int x^3 \cdot x^{-\frac{1}{3}}dx$$

$$= \int x^{3-\frac{1}{3}}dx = \int x^{\frac{8}{3}}dx = \frac{x^{\frac{8}{3}+1}}{\frac{8}{3}+1} + C$$

$$\frac{x^{\frac{11}{3}}}{\frac{11}{3}} + C = \frac{3}{11}\sqrt[3]{x^{11}} + C$$

④ هام

$$\int \sin^3(x)\cdot\sin 2x\,dx$$

لدينا $\sin 2x = 2\sin x\cdot\cos x$

$$\int \sin^3 x \cdot 2\sin x\cdot\cos x\cdot dx$$

$$\int 2\cdot\sin^4 x\cdot\cos x\,dx$$

$$2\int \underbrace{\cos x}_{g'}\cdot\underbrace{\sin^4 x}_{g^n}\,dx$$

$$2\,\frac{\sin^{4+1} x}{4+1} + C = \frac{2}{5}\sin^5 x + C$$

⑤ هام تعويض

$$\int \frac{x\,dx}{\sqrt{1+x}}$$

نضع $\sqrt{1+x} = u \Rightarrow \frac{1}{2\sqrt{1+x}}dx = du$

$$dx = 2\sqrt{1+x}\,du \Rightarrow dx = 2u\,du$$

$$\int \frac{x\cdot 2\cdot \cancel{u}\cdot du}{\cancel{u}} = \int 2x\,du$$

لدينا $\sqrt{1+x} = u \xrightarrow{\text{تربيع}} 1+x = u^2$

$$\Rightarrow x = u^2 - 1$$

(ب ج) نضع $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$dx = x\,du$

$$\int \frac{u^2}{\cancel{x}} \cdot \cancel{x} \cdot du = \int u^2\,du$$

$$= \frac{u^3}{3} + C = \frac{\ln^3 x}{3} + C$$

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(8) $\int \sqrt{\sin x} \cdot \cos x\,dx \quad]0, \pi[$

$$\int \underbrace{\sin^{\frac{1}{2}} x}_{g^n} \cdot \underbrace{\cos x\,dx}_{g'} = \frac{\sin^{\frac{1}{2}+1} x}{\frac{1}{2}+1} + C$$

$$\frac{\sin^{\frac{3}{2}} x}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{\sin^3 x} + C$$

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(9) $$\int \frac{\ln^5 x}{x}\,dx = \int \underbrace{\frac{1}{x}}_{g'} \underbrace{\ln^5 x}_{g^n}\,dx$$

$$= \frac{\ln^{5+1} x}{5+1} + C = \frac{1}{6}\ln^6 x + C$$

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(10) $$\int \tan 2x = \int \frac{\sin 2x}{\cos 2x}\,dx$$

$$= -\frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}\,dx = -\frac{1}{2}\int \frac{g'}{g}$$

$$= -\frac{1}{2}\ln|\cos 2x| + C$$

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(11) $$\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$$

$u = \frac{1}{x} \Rightarrow du = -\frac{1}{x^2}\,dx$

$dx = -x^2\,du$

$$\int \frac{1}{\cancel{x^2}} \cdot \sin u \cdot (-\cancel{x^2} \cdot du)$$

$$2\int (u^2 - 1)\,du = 2\left(\frac{u^3}{3} - u\right) + C$$ نبدل

نعود للمتحول الأصلي .

$$\frac{2\left(\sqrt{x+1}\right)^3}{3} - 2\sqrt{x+1} + C$$

$$= \frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1} + C$$

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(6) هام

$$\int \frac{x\,dx}{1+x^2}$$

نضع $u = 1 + x^2 \Rightarrow du = 2x\,dx$

$$\Rightarrow dx = \frac{du}{2x}$$

$$\int \frac{x \cdot \frac{du}{2x}}{u} = \int \frac{\frac{du}{2}}{\frac{u}{1}}$$

$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|1+x^2| + C$$

$$= \frac{1}{2}\ln(1+x^2) + C \Leftarrow 1+x^2 > 0 \text{ كون}$$

(ط ج) إذا لم يطلب بالتعويض ،

$$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= \frac{1}{2}\int \frac{g'}{g}\,dx = \frac{1}{2}\ln|1+x^2| + C$$

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(7) هام

$$\int \frac{(\ln x)^2}{x}\,dx$$

(ط ج)

$$\int \underbrace{\frac{1}{x}}_{g'} \cdot \underbrace{(\ln x)^2}_{g^n}\,dx = \frac{\ln^{2+1} x}{2+1} + C$$

$$= \frac{1}{3}\ln^3 x + C$$

$-\int \sin u \, du$

$= -(-\cos u) + C$

$= \cos u + C$

نعود للمتحول الأصلي.

$= \cos \frac{1}{x} + C$

ملاحظة: إذا كان داخل sin أو cos أي شيء عدا x أو ax نقوم بالتكامل بالتعويض بأن نأخذه u.

(١٢) $\int \frac{dx}{\sin^2 3x} = -\frac{1}{3} \int \underbrace{\frac{-3}{\sin^2 3x}}_{(\cot x)'} dx$

$= -\frac{1}{3} \cot x + C$

(١٣) $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$

ط١. $-\int \underbrace{-\frac{1}{x^2}}_{g'} \underbrace{e^{\frac{1}{x}}}_{e^{g}} dx = -e^{\frac{1}{x}} + C$

ط٢. $\frac{1}{x} = u \Rightarrow -\frac{1}{x^2} dx = du$

$dx = -x^2 du$

$\int \frac{1}{\cancel{x^2}} e^{u} \cdot (-\cancel{x^2} du) = -\int e^{u} du$

$= -e^{u} + C$

$= -e^{\frac{1}{x}} + C$

(١٤) $\int x \sin 2x \, dx$ تجزئة (هام)

$u = x \Rightarrow u' = 1 \; , \; v' = \sin 2x$

$\Rightarrow v = -\frac{1}{2} \cos 2x$

$u \cdot v - \int v \cdot u' dx$ قانون التكامل بالتجزئة.

$x \cdot (-\frac{1}{2} \cos 2x) - \int -\frac{1}{2} \cos 2x \, dx$

$= -\frac{1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x \, dx$

$= -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C$

(١٥) $\int x^2 e^{x} dx$

$u = x^2 \Rightarrow u' = 2x$

$v' = e^{x} \Rightarrow v = e^{x}$

$u \cdot v - \int v \cdot u' dx$

$x^2 e^{x} - \int 2x e^{x} dx$

$I_1 = \int 2x e^{x} dx$

$u = 2x \Rightarrow u' = 2$

$v' = e^{x} \Rightarrow v = e^{x}$

$I_1 = u \cdot v - \int v \cdot u' dx$

$= 2x e^{x} - \int e^{x} \cdot (2) \cdot dx$

$= 2x e^{x} - 2 e^{x}$

نبدل في (*) نجد

$I = x^2 e^{x} - (2x e^{x} - 2 e^{x}) + C$

$= x^2 e^{x} - 2x e^{x} + 2 e^{x} + C$

$= (x^2 - 2x + 2) e^{x} + C$

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٠٩٨٨٨٢١٦٤٧

(١٦)

$\int x\ln x\,dx$

$u=\ln x \Rightarrow u'=\frac{1}{x}$

$v'=x \Rightarrow v=\frac{x^2}{2}$

$u\cdot v-\int v\cdot u'\,dx$

$\frac{x^2}{2}\cdot\ln x-\int\frac{x^2}{2}\cdot\frac{1}{x}\,dx$

$\frac{x^2}{2}\ln x-\frac{1}{2}\int x\,dx$

$=\frac{x^2}{2}\ln x-\frac{1}{4}x^2+$ ث

(١٧)

$I=\int(x^2+1)\cos x\,dx$

$u=x^2+1\Rightarrow u'=2x$

$v'=\cos x\Rightarrow v=\sin x$

$u\cdot v-\int v\cdot u'\,dx$

$(x^2+1)\sin x-\int 2x\sin x\,dx$ ⊛

$I_1=\int 2x\sin x\,dx$

$u=2x\Rightarrow u'=2$

$v'=\sin x\Rightarrow v=-\cos x$

$I_1=u\cdot v-\int v\cdot u'\,dx$

$=-2x\cos x-\int -2\cos x\,dx$

$=-2x\cos x+2\int\cos x\,dx$

$=-2x\cos x+2\sin x$

نبدل في ⊛ نجد

$I=(x^2+1)\sin x+2x\cos x-2\sin x+C$

$=(x^2+1-2)\sin x+2x\cos x+C$

$=(x^2-1)\sin x+2x\cos x+C$

(١٨)

$I=\int 2x\cos^2 x\,dx$

لدينا

$\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$

$\int 2x\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx$

$\int(x+x\cos 2x)\,dx=\int x\,dx+\int x\cos 2x\,dx$

$I=\frac{x^2}{2}+\int x\cos 2x\,dx$ ⊛

$I_1=\int x\cos 2x\,dx$

$u=x\Rightarrow u'=1\ ,\ v'=\cos 2x\Rightarrow v=\frac{1}{2}\sin 2x$

$I_1=u\cdot v-\int v\cdot u'\,dx$

$I_1=\frac{1}{2}x\cdot\sin 2x-\frac{1}{2}\int\sin 2x\,dx$

$=\frac{1}{2}x\sin 2x-\frac{1}{2}\left(-\frac{1}{2}\cos 2x\right)$

$=\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x$

نبدل في ⊛ نجد

$I=\frac{x^2}{2}+\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x$ + ث

(١٩)

$\int e^{\sqrt{x}}\,dx$

$t=\sqrt{x}\Rightarrow dt=\frac{1}{2\sqrt{x}}\,dx$

$dt=\frac{1}{2t}\cdot dx\Rightarrow dx=2t\,dt$

$\int e^{t}\cdot 2t\cdot dt=\int 2te^{t}\,dt$

$\int 2te^{t}\,dt$ نكامل بالتجزئة

$u\cdot v - \int v\cdot u'\,dx$

$x\ln x - \int x\cdot\frac{1}{x}\,dx$

$x\ln x - \int 1\,dx = x\ln x - x$

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(٢٢) $\int \frac{1}{x\ln x}\,dx = \int \frac{\frac{1}{x}}{\ln x}\,dx$

$= \int \frac{g'}{g} = \ln|\ln x| + C$

$\ln x < 0 \Rightarrow$ إن $D_1\ ]0,1[$

$\ln(-\ln(x)) + C$

$\ln x > 0 \Rightarrow D_2\ ]1,+\infty[$

$\ln(\ln x) + C$

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(٢٣) $\int \frac{1}{\sin 2x}\,dx \xrightarrow{\text{على المجال}}]\frac{\pi}{2},\pi[$

$1 = C$ لدينا $\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\int \frac{\sin^2 x + \cos^2 x}{\sin 2x}\,dx = \int\left(\frac{\sin^2 x}{\sin 2x} + \frac{\cos^2 x}{\sin 2x}\right)dx$

$= \int \frac{\cancel{\sin^2 x}}{2\cancel{\sin x}\cos x}\,dx + \int \frac{\cos^2 x}{2\sin x\cos x}\,dx$

$\frac{1}{2}\int \frac{\sin x}{\cos x}\,dx + \frac{1}{2}\int \frac{\cos x}{\sin x}\,dx$

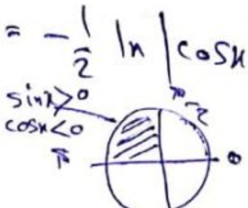
$= -\frac{1}{2}\int -\frac{\sin x}{\cos x}\,dx + \frac{1}{2}\int \frac{\cos x}{\sin x}\,dx$

$= -\frac{1}{2}\int \frac{g'}{g} + \frac{1}{2}\int \frac{g'}{g}$

$= -\frac{1}{2}\ln|\cos x| + \frac{1}{2}\ln|\sin x| + C$

$\sin x > 0$

$\cos x < 0$



$\Rightarrow -\frac{1}{2}\ln(-\cos x) + \frac{1}{2}\ln(\sin x)$

$= \frac{1}{2}\left[\ln(\sin x) - \ln(-\cos x)\right]$

$= \frac{1}{2}\ln\frac{\sin x}{-\cos x} = \frac{1}{2}\ln(-\tan x)$ $\boxed{C}$

$u = 2t \Rightarrow u' = 2$

$v' = e^t \Rightarrow v = e^t$

$u\cdot v - \int v\cdot u'\,dt$

$2t\cdot e^t - \int 2e^t\,dt$

$2t e^t - 2e^t + C$

نعود للمتحول الأصلي :

$2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

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(٢٠) $\int \frac{x}{\sqrt{2x+1}}\,dx = \int x(2x+1)^{-\frac{1}{2}}\,dx$

$u = x \Rightarrow u' = 1$

$v' = (2x+1)^{-\frac{1}{2}} \Rightarrow v = \cancel{\frac{1}{2}}\frac{(2x+1)^{-\frac{1}{2}+1}}{\cancel{-\frac{1}{2}+1}}$

$= (2x+1)^{\frac{1}{2}}$

$u\cdot v - \int v\cdot u'\,dx$

$x(2x+1)^{\frac{1}{2}} - \int (2x+1)^{\frac{1}{2}}\cdot dx$

$= x(2x+1)^{\frac{1}{2}} - \left[\frac{(2x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\times\frac{1}{2}\right]$

$x(2x+1)^{\frac{1}{2}} - \frac{2}{3}\times\frac{1}{2}\cdot(2x+1)^{\frac{3}{2}} + C$

$x(2x+1)^{\frac{1}{2}} - \frac{1}{3}(2x+1)^{\frac{3}{2}} + C$

$= x\sqrt{2x+1} - \frac{1}{3}\sqrt{(2x+1)^3} + C$

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(٢١) $\int \ln x\,dx$

$u = \ln x \Rightarrow u' = \frac{1}{x}$

$v' = 1 \Rightarrow v = x$

(٢٤)

$$\int_0^{\ln 2} \frac{1}{e^x+1}\, dx$$

لدينا $e^x \cdot e^{-x} = 1$

$$\int_0^{\ln 2} \frac{\cancel{e^x} \cdot e^{-x}}{\cancel{e^x}(1+\frac{1}{e^x})}\, dx = -\int_0^{\ln 2} \frac{-e^{-x}}{1+e^{-x}}\, dx$$

$$-\int_0^{\ln 2} \frac{g'}{g} = -\left[\ln(1+e^{-x})\right]_0^{\ln 2}$$

$$= -\left[\ln(1+e^{-\ln 2}) - \ln(1+e^{-0})\right]$$

$$= -\left[\ln(1+\tfrac{1}{2}) - \ln 2\right]$$

$$= -\left[\ln \tfrac{3}{2} - \ln 2\right] = -\ln 3 + \ln 2 + \ln 2$$

$$= -\ln 3 + 2\ln 2 = -\ln 3 + \ln 4$$

$$= \ln 4 - \ln 3 = \ln \frac{4}{3}$$

(٢٥)

$$\int_{-2}^{-1} \frac{2x-1}{x-1}\, dx$$

$$\int_{-2}^{-1} \frac{2x-1-1+1}{x-1}\, dx = \int_{-2}^{-1} \frac{2x-2+1}{x-1}\, dx$$

$$\int_{-2}^{-1} \frac{2(x-1)+1}{x-1}\, dx = \int_{-2}^{-1} \left(\frac{2\cancel{(x-1)}}{\cancel{x-1}} + \frac{1}{x-1}\right) dx$$

$$= \int_{-2}^{-1} \left(2 + \frac{1}{x-1}\right) dx = \left[2x + \ln|x-1|\right]_{-2}^{-1}$$

$$(-2+\ln 2) - (-4+\ln 3)$$

$$= -2 + \ln 2 + 4 - \ln 3$$

$$= 2 + \ln 2 - \ln 3$$

$$= 2 + \ln \frac{2}{3}$$

Nice

(٢٦)

$$\int_0^3 |2x+2|\, dx$$

ملاحظة: لكي نكامل الصيغة المطلقة نقوم أولاً بفك الصيغة المطلقة على مجال المكاملة ثم نكامل أي

$|\square| = \square$ عندما $\square > 0$

$|\square| = -\square$ عندما $\square < 0$

انتبه

نلاحظ: في المجال $[0,3]$ المقدار $2x+2>0$ وبالتالي

$$|2x+2| = 2x+2$$

$$\int_0^3 (2x+2)\, dx = \left[x^2+2x\right]_0^3$$

$$= (9+6) - (0+0) = 15$$

(٢٧) هام

$$\int_0^3 |x^2-3x+2|\, dx$$

أولاً نقوم بفك الصيغة المطلقة وبالتالي ندرس إشارة المقدار $x^2-3x+2=0$

$\Delta = 9 - 4\times1\times2 = 1$ $x_1 = \frac{3+1}{2} = 2$

$x_2 = \frac{3-1}{2} = 1$

x	$-\infty$	0		1		2		3	$+\infty$
x^2-3x+2	+	+		0	−	0	+	+	
		يوافق			يخالف			يوافق	

المقدار موجب $[0,1[$

المقدار سالب $]1,2[$

المقدار موجب $]2,3]$

$\Rightarrow$

$$\int_0^1 (x^2-3x+2)\, dx + \int_1^2 -(x^2-3x+2)\, dx$$

$$+ \int_2^3 (x^2-3x+2)\, dx$$

$$\int_{-2}^{+2}|x^2+4x|\,dx = \int_{-2}^{0} -(x^2+4x)\,dx + \int_{0}^{2}(x^2+4x)\,dx$$

$$= -\left[\frac{x^3}{3}+2x^2\right]_{-2}^{0} + \left[\frac{x^3}{3}+2x^2\right]_{0}^{2}$$

$$= -\left(0-\left(\frac{-8}{3}+8\right)\right)+\left[\left(\frac{8}{3}+8\right)-0\right]$$

$$= -\cancel{\frac{8}{3}}+8+\cancel{\frac{8}{3}}+8 = 16$$

(29)

$$\int_{0}^{\frac{\pi}{4}}\frac{\sin^3 x}{\cos^5 x}\,dx = \int_{0}^{\frac{\pi}{4}}\frac{\sin^3 x}{\cos^3 x\cdot\cos^2 x}\,dx$$

$$= \int_{0}^{\frac{\pi}{4}}\frac{\sin^3 x}{\cos^3 x}\cdot\frac{1}{\cos^2 x}\,dx = \int_{0}^{\frac{\pi}{4}}\left(\frac{\sin x}{\cos x}\right)^3\cdot\frac{1}{\cos^2 x}\,dx$$

$$= \int_{0}^{\frac{\pi}{4}}\underbrace{\tan^3 x}_{g^n}\cdot\underbrace{\frac{1}{\cos^2 x}}_{g'}\,dx$$

كما أن $(\tan x)' = \frac{1}{\cos^2 x}$ واو واوووو

إذن

$$= \left[\frac{\tan^{3+1} x}{3+1}\right]_{0}^{\frac{\pi}{4}}$$

$$= \left[\frac{1}{4}\tan^4 x\right]_{0}^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{4}(1)^4\right)-\left(\frac{1}{4}(0)^4\right)$$

$$= \frac{1}{4}$$

(30)

$$\int x\cdot\sqrt{4-x^2}\,dx$$

$$\int x(4-x^2)^{\frac{1}{2}}\,dx$$

$$= \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_{0}^{1} - \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_{1}^{2} + \left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_{2}^{3}$$

$$= \left[\left(\frac{1}{3}-\frac{3}{2}+2\right)-0\right]-\left[\left(\frac{8}{3}-\frac{12}{2}+4\right)-\left(\frac{1}{3}-\frac{3}{2}+2\right)\right] + \left[\left(\frac{27}{3}-\frac{27}{2}+6\right)-\left(\frac{8}{3}-\frac{12}{2}+4\right)\right]$$

$$= \left[\left(\frac{1}{3}-\frac{3}{2}+2\right)-\left(\frac{8}{3}-6+4\right)+\left(\frac{1}{3}-\frac{3}{2}+2\right)+\left(\frac{27}{3}-\frac{27}{2}+6\right)-\left(\frac{8}{3}-6+4\right)\right]$$

$$= \left[\frac{2-9+12}{6}-\left(\frac{8}{3}-2\right)+\left(\frac{2-9+12}{6}\right)+\left(15-\frac{27}{2}\right)-\left(\frac{8}{3}-2\right)\right]$$

$$= \ldots - \frac{2}{3} + \frac{5}{6} + \frac{3}{2} - \frac{2}{3}$$

2   3   2

$$= \frac{5-4+5+9-4}{6} = \frac{19-8}{6} = \frac{11}{6}$$

(28)

$$\int_{-2}^{+2}|x^2+4x|\,dx$$

ندرس إشارة المقدار x^2+4x لنتخلص من القيمة المطلقة

$$x^2+4x=0 \Rightarrow x(x+4)=0$$

إما $x=0$ أو $x=-4$

$-\infty$	-4	-2	0	$+2$	$+\infty$
$+$		$-$		$+$	

$[-2,0[$ $x^2+4x<0$

$]0,2]$ $x^2+4x>0$

$\int x \cdot \sin t \cdot \frac{dt}{\cancel{2x}}$

$\frac{1}{2}\int \sin t \, dt = \frac{1}{2}(-\cos t) + C$

$= -\frac{1}{2}\cos t + C$

بالعودة للمتحول الأصلي.

$-\frac{1}{2}\cos x^2 + C$

---

(٣٣) حل

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x} \, dx$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)} \, dx$

لدينا

$\sin^2 x + \cos^2 x = 1 \Rightarrow$

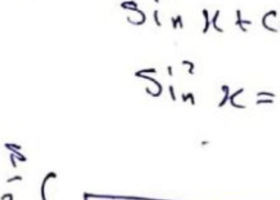
$\sin^2 x = 1 - \cos^2 x$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x \cdot \sin^2 x} \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x \cdot \sin^2 x)^{\frac{1}{2}} dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^{\frac{1}{2}} x \cdot (\sin^2 x)^{\frac{1}{2}} dx$

تذكر: $\sqrt{x^2} = |x|$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^{\frac{1}{2}} x \cdot |\sin x| \, dx \Rightarrow$



$\int_{-\frac{\pi}{2}}^{0} -\sin x \cdot \cos^{\frac{1}{2}} x \, dx$

$+ \int_{0}^{\frac{\pi}{2}} \sin x \cdot \cos^{\frac{1}{2}} x \, dx$

$= \int_{-\frac{\pi}{2}}^{0} \underbrace{-\sin x}_{g'} \cdot \underbrace{\cos^{\frac{1}{2}} x}_{g^n} dx - \int_{0}^{\frac{\pi}{2}} \underbrace{-\sin x}_{g'} \cdot \underbrace{\cos^{\frac{1}{2}} x}_{g^n} dx$

$-\frac{1}{2}\int -2x(4-x^2)^{\frac{1}{2}} dx$

$= -\frac{1}{2}\int g' \cdot g^n$

$= -\frac{1}{2}\left[\frac{(4-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$

$-\frac{1}{2} \cdot \frac{2}{3}(4-x^2)^{\frac{3}{2}} + C$

$= -\frac{1}{3}\sqrt{(4-x^2)^3} + C$

---

(٣١)

$\int_0^9 \frac{x-1}{1+\sqrt{x}} dx$

لدينا: $x - 1 = (\sqrt{x})^2 - (1)^2 = (\sqrt{x}+1)(\sqrt{x}-1)$

$\int_0^9 \frac{\cancel{(\sqrt{x}+1)}(\sqrt{x}-1)}{\cancel{(\sqrt{x}+1)}} dx$

$= \int_0^9 (\sqrt{x}-1) dx = \int_0^9 (x^{\frac{1}{2}}-1) dx$

$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - x\right]_0^9 = \left[\frac{2}{3}\sqrt{x^3} - x\right]_0^9$

$\left(\frac{2}{3}(27) - 9\right) - \left(\frac{2}{3}(0) - 0\right)$

$18 - 9 = 9$

---

(٣٢)

$\int x \sin x^2 \, dx$

$x^2 = t \Rightarrow 2x\,dx = dt$

$dx = \frac{dt}{2x}$

$$= \left[- \frac{(1-x)^{\frac{4}{3}}}{\frac{4}{3}} \right]_1^2 = \left[-\frac{3}{4}\sqrt[3]{(1-x)^4} \right]_1^2$$

$$= \left(-\frac{3}{4}\sqrt[3]{(1-2)^4}\right) - \left(-\frac{3}{4}\sqrt[3]{(1-1)^4}\right)$$

$$= -\frac{3}{4}$$

(٣٦)

$$\int \frac{8}{x^3(x-1)}\,dx$$

$$\frac{8}{x^3(x-1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x-1}$$

لتحديد A نضرب بـ x^3 و نجعل $x \longleftarrow 0$

$$\frac{8}{x-1}\Big|_{x=0} = A \Rightarrow A = \frac{8}{-1} \Rightarrow \boxed{A=-8}$$

لتحديد D نضرب بـ $x-1$ و نجعل $x \longleftarrow 1$

$$\frac{8}{x^3}\Big|_{x=1} = D \Rightarrow D = \frac{8}{1} \Rightarrow \boxed{D=8}$$

$$\frac{8}{x^3(x-1)} = \frac{-8}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{8}{x-1}$$

لتحديد B و C . نضع مثلاً $x=-1$ نجد

$$4 = 8 + B - C - 4 \Rightarrow \boxed{B-C=0} \quad (1)$$

نضع مثلاً $x=2$ نجد

$$-1 + \frac{B}{4} + \frac{C}{2} + 8$$

$$\frac{B}{4} + \frac{C}{2} = -6 \Rightarrow \boxed{B+2C=-24} \quad (2)$$

نضرب (1) بـ $\boxed{-1}$ ونجمع مع (2) نجد

$$= -24 \Rightarrow \boxed{C=-8}$$

نعوض في (1) نجد

$$B+8=0 \Rightarrow \boxed{B=-8}$$

$$\frac{8}{x^3(x-1)} = \frac{-8}{x^3} + \frac{-8}{x^2} + \frac{-8}{x} + \frac{8}{x-1}$$

$$= -8x^{-3} - 8x^{-2} - \frac{8}{x} + \frac{8}{x-1}$$

$$\Rightarrow \int \frac{8}{x^3(x-1)}\,dx = \int -8x^{-3}\,dx - \int 8x^{-2}\,dx - 8\int \frac{1}{x}\,dx + 8\int \frac{1}{x-1}\,dx$$

$$\left[\frac{\cos^{\frac{1}{2}+1} x}{\frac{1}{2}+1} \right]_{-\frac{\pi}{2}}^{0} + \left[\frac{\cos^{\frac{1}{2}+1} x}{\frac{1}{2}+1} \right]_{0}^{\frac{\pi}{2}}$$

$$= \left[\frac{2}{3}\sqrt{\cos^3 x} \right]_{-\frac{\pi}{2}}^{0} - \left[\frac{2}{3}\sqrt{\cos^3 x} \right]_{0}^{\frac{\pi}{2}}$$

$$= \left[\frac{2}{3}(1) - \frac{2}{3}(0)\right] - \left[\frac{2}{3}(0) - \frac{2}{3}(1)\right]$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

(٣٤)

$$\int_1^{e^3} \frac{1}{x\sqrt{1+\ln x}}\,dx$$

$$= \int_1^{e^3} \frac{1}{x(1+\ln x)^{\frac{1}{2}}}\,dx = \int_1^{e^3} \underbrace{\frac{1}{x}}_{g'} \cdot \underbrace{(1+\ln x)^{-\frac{1}{2}}}_{g^n}\,dx$$

$$\left[\frac{(1+\ln x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_1^{e^3}$$

$$\left[\frac{(1+\ln x)^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^{e^3} = \left[2(1+\ln x)^{\frac{1}{2}} \right]_1^{e^3}$$

$$= \left[2\sqrt{1+\ln x} \right]_1^{e^3}$$

$$= \left(2\sqrt{1+\ln e^3}\right) - \left(2\sqrt{1+\ln 1}\right)$$

$$= 2\sqrt{1+3\ln e} - 2\sqrt{1+0}$$

$$= 2\sqrt{1+3} - 2 = 2(2) - 2 = (٢)$$

(٣٥)

$$\int_1^2 \sqrt[3]{(1-x)}\,dx$$

$$= \int_1^2 (1-x)^{\frac{1}{3}}\,dx = -\frac{(1-x)^{\frac{1}{3}+1}}{\frac{1}{3}+1}$$

$= -8\frac{x^{-2}}{-2} - 8\frac{x^{-1}}{-1} - 8\ln|x| + 8\ln|x-1| + C$

$= \frac{4}{x^2} + \frac{8}{x} + 8\ln|x-1| - 8\ln|x| + C$

$= \frac{4}{x^2} + \frac{8}{x} + 8\left(\ln|x-1| - \ln|x|\right) + C$

$= \frac{4}{x^2} + \frac{8}{x} + 8\ln\frac{|x-1|}{|x|} +$

٣٧

$\int \frac{x^2+x+2}{(x-1)(x^2+1)}\,dx$

$\frac{x^2+x+2}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$

لتحديد A نضرب بـ $x-1$ ونجعل $x \to 1$

$\left.\frac{x^2+x+2}{x^2+1}\right|_{x=1} = A = \frac{4}{2} \Rightarrow \boxed{A=2}$

لتحديد B نضرب بـ x ونجعل $x \to \infty$

$\frac{x^3+x^2+2x}{x^3-x^2+x-1} = \frac{2x}{x-1} + \frac{Bx^2+Cx}{x^2+1}$

$1 = 2 + B \Rightarrow \boxed{B=-1}$

لتحديد C نضع مثلاً $x=0$ نجد

$\frac{2}{-1} = \frac{2}{-1} + \frac{C}{1}$

$-2 = -2 + C \Rightarrow \boxed{C=0}$

$\int \frac{x^2+x+2}{(x-1)(x^2+1)}\,dx = \int\left(\frac{2}{x-1} + \frac{-x}{x^2+1}\right)dx$

$= 2\ln|x-1| - \frac{1}{2}\ln|x^2+1| + C$

$\ln(x-1)^2 - \ln\sqrt{x^2+1} + C$

$= \ln\frac{(x-1)^2}{\sqrt{x^2+1}} +$

٣٨

$\int 2\cos 3x\cdot\sin 2x\,dx$

$\int 2\cdot\frac{1}{2}(\sin 5x - \sin x)\,dx$

$= \int(\sin 5x - \sin x)\,dx$

$= -\frac{1}{5}\cos 5x + \cos x +$

٣٩

$\int \cos 2x\cdot\cos x\,dx$

$= \int \frac{1}{2}(\cos 3x + \cos x)\,dx$

$= \frac{1}{2}\cdot\frac{1}{3}\sin 3x + \frac{1}{2}\sin x + C$

$= \frac{1}{6}\sin 3x + \frac{1}{2}\sin x +$

٤٠

$\int \ln^2 x\,dx$

نكامل بالتجزئة.

$u = \ln^2 x \Rightarrow u' = 2\cdot\frac{1}{x}\cdot\ln x$

$v' = 1 \Rightarrow v = x$

$u.v - \int v.u'\,dx$

$x.\ln^2 x - \int \cancel{x}.2.\frac{1}{\cancel{x}}\cdot\ln x$

$x\ln^2 x - 2\int \ln x\,dx$

كاملناه سابقاً بالتجزئة.

$x\ln^2 x - 2(x\ln x - x)$

$x\ln^2 x - 2x\ln x + 2x +$

٤١

$\int_0^1 \frac{e^x}{e^x+1}\,dx = \int_0^1 \frac{g'}{g}\,dx$

$\left[\ln(e^x+1)\right]_0^1 = \ln(e+1) - \ln 2$

$= \ln\frac{e+1}{2}$

(٤٢)

$$I=\int x(1+x)^n\,dx \qquad n\in N^*$$

$$\int (x+1-1)(1+x)^n\,dx$$

$$\int \left((x+1)(1+x)^n-(1+x)^n\right)dx$$

$$\int \left((1+x)^{n+1}-(1+x)^n\right)dx$$

$$=\frac{(1+x)^{n+1+1}}{n+1+1}\times\frac{1}{1}-\frac{(1+x)^{n+1}}{n+1}\times\frac{1}{1}+C$$

$$=\frac{(1+x)^{n+2}}{n+2}-\frac{(1+x)^{n+1}}{n+1}+C$$

(٤٣)

$$\int_0^{\frac{\pi}{2}}\sin^2 x\,dx$$

لدينا: $\sin^2 x=\frac{1}{2}-\frac{1}{2}\cos 2x$

$$\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$$

$$=\left[\frac{1}{2}x-\frac{1}{2}\cdot\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$=\left(\frac{\pi}{4}-\frac{1}{4}\sin\pi\right)-\left(0-\frac{1}{4}\sin 0\right)$$

$$=\frac{\pi}{4}-\frac{1}{4}(0)=\frac{\pi}{4}$$

(٤٤)

$$\int_{-1}^{0}\frac{-2x}{(x-1)^2}\,dx$$

$x-1=u \Rightarrow dx=du$

$x=u+1$

$x=0\Rightarrow u=-1$

$x=-1\Rightarrow u=-2$

$$\int_{-2}^{-1}\frac{-2(u+1)}{u^2}\cdot du$$

$$-2\int_{-2}^{-1}\left(\frac{u}{u^2}+\frac{1}{u^2}\right)du=-2\int_{-2}^{-1}\left(\frac{1}{u}+u^{-2}\right)du$$

$$=-2\left[\ln|u|+\frac{u^{-1}}{-1}\right]_{-2}^{-1}$$

$$=-2\left[\ln|u|-\frac{1}{u}\right]_{-2}^{-1}$$

$$=-2\left[(\ln 1+1)-\left(\ln 2+\frac{1}{2}\right)\right]$$

$$=-2\left[+1-\ln 2-\frac{1}{2}\right]$$

$$=-2\left[\frac{1}{2}-\ln 2\right]=-1+2\ln 2$$

$$=2\ln 2-1=\ln 4-1$$

(٤٥)

$$\int_0^{\ln 2}\frac{e^x-1}{e^x+1}\,dx=\int_0^{\ln 2}\frac{e^x+1-1-1}{e^x+1}\,dx$$

$$\int_0^{\ln 2}\frac{e^x+1-2}{e^x+1}\,dx=\int_0^{\ln 2}\left(\frac{e^x+1}{e^x+1}-\frac{2}{e^x+1}\right)dx$$

$$=\int_0^{\ln 2}\left(1-\frac{2}{e^x+1}\right)dx=\int_0^{\ln 2}\left(1-2\frac{1}{e^x+1}\right)dx$$

$$\int_0^{\ln 2}1\,dx-2\int_0^{\ln 2}\frac{1}{e^x+1}\,dx$$

$$x-2\int_0^{\ln 2}\frac{\cancel{e^x}\cdot e^{-x}}{\cancel{e^x}(1+e^{-x})}\,dx$$

$$x+2\int_0^{\ln 2}\frac{-e^{-x}}{1+e^{-x}}\,dx=\left[x+2\ln(1+e^{-x})\right]_0^{\ln 2}$$

$$=\ln 2+2\ln\left(1+\frac{1}{2}\right)-\left(0+2\ln(2)\right)$$

$$=\ln 2+2\ln\frac{3}{2}-2\ln 2=\ln 2+2\ln 3-2\ln 2-2\ln 2$$

$$=2\ln 3-3\ln 2$$

$$=\ln\frac{9}{8}$$

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