

## حلول اختبار العقديّة وتطبيقاتها (5+4)

### السؤال الأول :

$$z = -(\sqrt{3}-1) \left( \cos\left(\frac{\pi}{2}-\frac{\pi}{5}\right) + i \sin\left(\frac{\pi}{2}-\frac{\pi}{5}\right) \right)$$

$$z = (\sqrt{3}-1)e^{i\pi} \cdot e^{i\frac{3\pi}{10}} = (\sqrt{3}-1)e^{i\frac{13\pi}{10}}$$

### السؤال الرابع :

$$(x+yi+3)(x-yi-4)$$

$$x^2 - xyi - 4x + yxi + y^2 - 4yi + 3x - 3yi - 12$$

$$(x^2 - 4x + y^2 + 3x - 12) + i(-4y - 3y)$$

$$-4y - 3y = 0$$

$$-7y = 0 \Rightarrow y = 0$$

جميع النقاط في الفواصل :

### السؤال الثاني :

$$\bar{z} = \frac{z - z \cdot \bar{w}}{-i + \bar{w}i}$$

نضرب ب:  $w$  :

السط والمقام

$$\bar{z} = \frac{w \cdot z - z \cdot \bar{w} \cdot w}{-i w + w \cdot \bar{w} \cdot i}$$

$$w \bar{w} = |w|^2 = 1$$

$$\bar{z} = \frac{w \cdot z - z}{i - iw} = -\frac{(z - w \bar{z})}{i - iw}$$

$$= -z$$

### السؤال الثالث :

نصف دائرة  $c$  : مركزه  $A$  : زاوية  $\frac{\pi}{4}$

$$z - a = e^{i\frac{\pi}{4}}(z - a)$$

$$b - a = i(c - a)$$

$$b - a = ic - ia$$

$$-a + ia = -b + ic$$

$$a - ia = b - ic$$

$$a(1-i) = b - ic$$

$$a = \frac{(b - ic)(1+i)}{(1-i)(1+i)} = \frac{b(1+i) - ic(1+i)}{2}$$

$$a = \frac{1}{2} [b(1+i) + c(1-i)]$$

### السؤال الخامس :

$$z_{OA} = a - 0 = 3 + i$$

-1

$$z_{OB} = b - 0 = 1 + 2i$$

$$z_{OA} = \sqrt{10} e^{i\alpha}, \quad z_{OB} = \sqrt{5} e^{i\beta}$$

-2

$$\frac{z_{OB}}{z_{OA}} = \frac{1+2i}{3+i} = \frac{(1+2i)(3-i)}{10}$$

$$= \frac{3-i+6i+2}{10} = \frac{5+5i}{10}$$

$$= \frac{1}{2}(1+i) = \frac{1}{2}\sqrt{2} e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{2} e^{i\frac{\pi}{4}}$$

$$\arg\left(\frac{z_{OB}}{z_{OA}}\right) = (\angle A, \angle B) = \frac{\pi}{4}$$

-3

بوابل عزيزان

