أوجد نهاية كل من التوابع التالية عند قيمة المطاة ...

1) $f(x) = -3x^2 + 4x + 1$ — $\mp\infty$

2) $f(x) = 1 - 3x$ — $\mp\infty$

3) $f(x) = x^2 - x$ — $\mp\infty$

4) $f(x) = \sqrt{x^2 + 4x + 3}$ — $-\infty$

5) $f(x) = \sqrt{\frac{9x-1}{3x+2}}$ — $+\infty$

6) $f(x) = \frac{1}{\sqrt{x-1}}$ — $+\infty$, 1

7) $f(x) = \frac{x-2}{(x-3)^2}$ — $+\infty$, 3

8) $f(x) = \frac{x-1}{x-2}$ — $+\infty$, 2

9) $f(x) = \frac{x}{x^2-x-2}$ — $+\infty$, -1

10) $f(x) = \frac{x+3}{x^2-x}$ — $-\infty$, 0

11) $f(x) = \frac{x}{x^2+1}$ — $+\infty$, 1

12) $f(x) = \frac{x^3-1}{x^2-3x+2}$ — 1

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

13) $f(x) = \frac{\sqrt{2x^3-1} - 1}{x-1}$ — 1

14) $f(x) = \sqrt{2x^2+3} - \sqrt{2x^2-5}$ — $+\infty$

15) $f(x) = \frac{\sqrt{5-x} - 2}{x-1}$ — 1

16) $f(x) = \frac{x-1}{\sqrt{2x} - \sqrt{3-x}}$ — 1

17) $f(x) = \frac{2-x}{\sqrt{7+6x^2}}$ — $-\infty$

18) $f(x) = \frac{\sqrt{x}}{1+2x+x^2}$ — 4

19) $f(x) = \frac{x(\sqrt{x}+1)}{x^2+1}$ — $+\infty$

20) $f(x) = \frac{-2x+1}{\sqrt{x^2+1}}$ — $-\infty$

21) $f(x) = \frac{\sqrt{2x+10} - 4}{x-3}$ — 3

22) $f(x) = \frac{\sqrt{4x^2+5}}{1+3x}$ — $-\infty$

23) $f(x) = \sqrt{x^2+1} - x$ — $+\infty$

24) $f(x) = 2x+1 - \sqrt{x^2+x-2}$ — $+\infty$

25) $f(x) = x - \sqrt{x}$ — $+\infty$

26) $f(x) = \sqrt[3]{x^2+5x+3}$ — 1

27) $f(x) = \sqrt{\frac{x^2-2x+1}{1+2x}}$ — 1

28) $f(x) = 2\sqrt{x} - \sqrt{4x+1}$ $\quad +\infty$

29) $f(x) = \dfrac{3x^5 - 2x + 1}{x^3 - 1}$ $\quad -\infty$

30) $f(x) = \sqrt{x} - 2x + 1$ $\quad +\infty$

31) $f(x) = 9$ $\quad 8$

32) $f(x) = -5x$ $\quad -\infty$

33) $f(x) = \dfrac{6 - x^3}{7x^3 + 1}$ $\quad +\infty$

34) $f(x) = \dfrac{x^3 - 1}{x^2 - 1}$ $\quad 1, -1$

35) $f(x) = \dfrac{\sqrt{4-x} - 2}{\sqrt{x+1} - 1}$ $\quad 0$

36) $f(x) = \dfrac{x^2 - 4}{|x-2|}$ $\quad 2$

37) $f(x) = \dfrac{\sqrt{x} - 1}{x^2 - 1}$ $\quad 1$

38) $f(x) = \dfrac{|x^3 - 4x|}{x+2}$ $\quad -2$

39) $f(x) = \dfrac{\sqrt{x^2 - 1}}{\sqrt{x^2 + 1}}$ $\quad -\infty$

40) $f(x) = \dfrac{\sin^2 x}{3x^2}$ $\quad 0$

41) $f(x) = \dfrac{\cos x - 1}{x}$ $\quad 0$

42) $f(x) = \dfrac{\cos x - 1}{\sin x}$ $\quad 0$

43) $f(x) = \dfrac{\sqrt{x+1} - 1}{\tan x}$ $\quad 0$

44) $f(x) = \dfrac{\cos x - \sin x}{x - \frac{\pi}{4}}$ $\quad \frac{\pi}{4}$

45) $f(x) = \dfrac{x - 2x^2 + x^3}{\cos(2x-2) - 1}$ $\quad 1$

46) $f(x) = \dfrac{\sin(x-1)}{x^2 - 1}$ $\quad 1$

47) $f(x) = \dfrac{\sqrt{2x^3 - 1} - 1}{x^2 - 1}$ $\quad 1$

48) $f(x) = \dfrac{x \cdot \cos x}{x^2 + 1}$ $\quad +\infty$

49) $f(x) = \dfrac{\tan 3x - x}{x + 2\sin x}$ $\quad 0$

50) $f(x) = \dfrac{\sin 2x}{\sqrt{x+1} - 1}$ $\quad 0$

51) $f(x) = \dfrac{\sin 2x}{\sqrt{2x+3} - \sqrt{3}}$ $\quad 0$

52) $f(x) = \dfrac{\sqrt{1 + \sin 2x} - \sqrt{1 - \sin 2x}}{x}$ $\quad 0$

53) $f(x) = \dfrac{\sqrt{1+x^2} - \cos x}{x^2}$ $\quad 0$

55) $f(x) = \dfrac{\sqrt{1 + \sin^2 x} - \cos x}{\sin^2 x}$ $\quad 0$

56) $f(x)=\dfrac{1-\cos x+\sin x}{x}$; 0

57) $f(x)=\dfrac{1}{\sqrt{x}}\sin x$; 0

58) $f(x)=\dfrac{\sin x-1}{x-\frac{\pi}{2}}$; $\frac{\pi}{2}$

59) $f(x)=\dfrac{\sqrt{x+9}-3}{\sin 7x}$; 0

60) $f(x)=x\cdot\sin\dfrac{1}{x}$; $+\infty$

61) $f(x)=\dfrac{\cos x}{x-\frac{\pi}{2}}$; $\frac{\pi}{2}$

62) $f(x)=\dfrac{1-\sqrt{\cos x}}{\tan^2 x}$; 0

63) $f(x)=\dfrac{x+\tan x}{x+\sin x}$; 0

64) $f(x)=\dfrac{x\cdot\sin x}{\sin^2 5x}$; 0

65) $f(x)=\dfrac{x^2}{\sin^2 3x}$; 0

66) $f(x)=\dfrac{\sin 3x}{\sin 4x}$; 0

67) $f(x)=\dfrac{\sin x-2x}{x-2\sin x}$; 0

68) $f(x)=\dfrac{4x}{\sin 2x}$; 0

69) $f(x)=\dfrac{\sin(\sin 2x)}{\sin 5x}$; 0

70) $f(x)=\dfrac{\sin x+\sin 2x+\sin 3x}{\sin 4x+\sin 5x+\sin 6x}$; 0

71) $f(x)=\dfrac{\sin x-\sin 3x}{x}$; 0

72) $f(x)=\dfrac{x+\sin x}{2x-3\sin 2x}$; 0

73) $f(x)=\dfrac{\tan(mx)}{\sin(nx)}$; 0

74) $f(x)=\dfrac{x\cdot\sin x}{1-\cos^2 2x}$; 0

75) $f(x)=\dfrac{\sin^2 x-x^2}{3x^2}$; 0

76) $f(x)=\dfrac{3\sin 2x}{-5x}$; 0

77) $f(x)=\dfrac{1-\cos^2 x}{2x^2}$; 0

78) $f(x)=\dfrac{\sin(2025\,)}{\sin(2026x}$; 0

79) $f(x)=\cos\left(\pi\,\dfrac{\sin x}{x}\right)$; 0

80) $f(x)=\dfrac{\sqrt{1-\cos x}}{\tan x}$; 0

81) $f(x)=\dfrac{\sin(x^2-x)}{x-1}$; 1

82) $f(x)=\dfrac{\sin x-\cos x}{x-\frac{\pi}{4}}$ $\frac{\sqrt{2}}{1}$

83) $f(x)=\dfrac{\tan x-\sin x}{x^3}$ 0

84) $f(x)=\dfrac{\sin(\pi x)}{x-1}$ π

85) $f(x)=\dfrac{\cos x}{\cos(2x)}$ $\frac{\sqrt{2}}{2}$

86) $f(x)=\dfrac{\sin(2x)-2\sin x}{x^3}$ 0

87) $f(x)=\dfrac{2\sin x-1}{6x-\pi}$ $\frac{\sqrt{3}}{9}$

88) $f(x)=\dfrac{\sin(2x)}{\sqrt{1-\cos x}}$ 0

89) $f(x)=\dfrac{\sin 2x}{\sqrt{x+1}-1}$ 0

90) $f(x)=\dfrac{\sin^2 3x-x^2}{1-\cos 2x}$ 0

91) $f(x)=\dfrac{\cos x}{\sqrt[3]{1-\sin x}}$ $\frac{\sqrt{3}}{2}$

92) $f(x)=\dfrac{x^2-1+\cos^2 x}{x\cdot\sin x}$ 0

93) $f(x)=\cos\left(\dfrac{\pi x-1}{x-2}\right)$ $+\infty$

94) $f(x)=\sin\left(\dfrac{\pi x+1}{2x+3}\right)$ $+\infty$

95) $f(x)=\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ 0

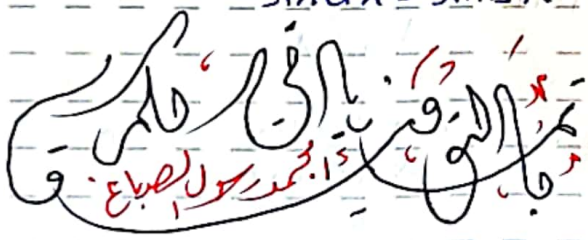
96) $f(x)=\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$ 0

97) $f(x)=\dfrac{1-\cos 4x}{x^2}$ 0

98) $f(x)=\dfrac{x\cdot\sin 3x}{1-\cos x}$ 0

99) $f(x)=\dfrac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$ 0

100) $f(x)=\dfrac{(a-b)x}{\sin ax-\sin bx}$ 0



أ) $\lim_{x\to\pm\infty} f(x) = -\infty$

2) $\lim_{x\to\mp\infty} f(x) = \pm\infty$

3) $\lim_{x\to\pm\infty} f(x) = +\infty$

4) $\lim_{x\to-\infty} f(x) = +\infty$

5) $\lim_{x\to+\infty} f(x) = \sqrt{\frac{9}{3}} = \sqrt{3}$

6) $\lim_{x\to+\infty} f(x) = 0$ $\quad \lim_{x\to1} f(x) = \frac{1}{0^+} = +\infty$

7) $\lim_{x\to+\infty} f(x) = 0$ $\quad \lim_{x\to3} f(x) = \frac{1}{0^+} = +\infty$

8) $\lim_{x\to+\infty} f(x) = 1$ $\quad \lim_{x\to2^+} f(x) = \frac{1}{0^+} = +\infty$ $\quad \lim_{x\to2^-} f(x) = \frac{1}{0^-} = -\infty$

9) $\lim_{x\to+\infty} f(x) = 0$ $\quad \lim_{x\to-1^+} f(x) = \frac{-1}{0^-} = +\infty$

$-\infty$	-1	2	$+\infty$
$+$	0	$-$	0 $+$

$\lim_{x\to-1^-} f(x) = \frac{-1}{0^+} = -\infty$

10) $\lim_{x\to+\infty} f(x) = 0$ $\quad \lim_{x\to0^+} f(x) = \frac{3}{0^-} = -\infty$

$-\infty$	0	1	$+\infty$
$+$	0	$-$	0 $+$

$\lim_{x\to0^-} f(x) = \frac{3}{0^+} = +\infty$

11) $\lim_{x\to+\infty} f(x) = 0$ $\quad \lim_{x\to1} f(x) = \frac{1}{2}$

12) $\lim_{x\to1} f(x) = \frac{0}{0}$ عدم تعيين

$$\lim_{x\to1} \frac{(x-1)(x^2+x+1)}{(x-2)(x-1)} = \lim_{x\to1} \frac{x^2+x+1}{x-2} = -3$$

13) $\lim_{x\to1} f(x) = \frac{0}{0}$ عدم تعيين

$$\lim_{x\to1} \frac{2x^3-1-1 = 2x^3-2}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$\lim_{x\to1} \frac{2(x^3-1) = 2(x-1)(x^2+x+1)}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$= \frac{2(1+1+1)}{1+1} = 3$$

14) $\lim_{x\to+\infty} f(x) = \infty - \infty$ عدم تعيين

$$\lim_{x\to+\infty} \frac{2x^2+3-2x^2+5}{\sqrt{2x^2+3}+\sqrt{2x^2-5}} = 0$$

15) $\lim_{x\to1} f(x) = \frac{0}{0}$ عدم تعيين

$$\lim_{x\to1} \frac{5-x-4}{(x-1)(\sqrt{5-x}+2)} = \lim_{x\to1} \frac{1-x}{(x-1)(\sqrt{5-x}+2)}$$

$$= \frac{-1}{2+2} = \frac{-1}{4}$$

16) $\lim_{x\to1} f(x) = \frac{0}{0}$ عدم تعيين

$$\lim_{x\to1} \frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{2x-3+x = 3x-3}$$

$$\lim_{x\to1} \frac{(x-1)(\sqrt{2x}+\sqrt{3-x})}{3(x-1)}$$

$$= \frac{\sqrt{2}+\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

17) $f(x)=\dfrac{2-x}{\sqrt{7+6x^2}}$ $\quad -\infty$

$\lim\limits_{x\to-\infty} f(x)=\dfrac{\infty}{\infty}$ عدم تعيين

$\lim\limits_{x\to-\infty}\dfrac{x(\frac{2}{x}-1)}{|x|\sqrt{\frac{7}{x^2}+6}}=\lim\limits_{x\to-\infty}\dfrac{x(\frac{2}{x}-1)}{-x\sqrt{\frac{7}{x^2}+6}}$

$=\dfrac{-1}{-\sqrt{6}}=\dfrac{1}{\sqrt{6}}$

$|x|=\begin{cases} x & +\infty \\ -x & -\infty \end{cases}$

18) $f(x)=\dfrac{\sqrt{x}}{1+2x+x^2}$ $\quad 4$

$\lim\limits_{x\to4} f(x)=\dfrac{2}{25}$

19) $f(x)=\dfrac{x(\sqrt{x}+1)}{x^2+1}$ $\quad +\infty$

$\lim\limits_{x\to+\infty} f(x)=0$ درجة البسط أكبر من درجة المقام بواحد

20) $f(x)=\dfrac{-2x+1}{\sqrt{x^2+1}}$ $\quad -\infty$

$\lim\limits_{x\to-\infty} f(x)=\dfrac{\infty}{\infty}$ عدم تعيين

$\lim\limits_{x\to-\infty}\dfrac{x(-2+\frac{1}{x})}{-x\sqrt{1+\frac{1}{x^2}}}=\dfrac{-2}{-1}=2$

21) $f(x)=\dfrac{\sqrt{2x+10}-4}{x-3}$ $\quad 3$

$\lim\limits_{x\to3} f(x)=\dfrac{0}{0}$ عدم تعيين

$\lim\limits_{x\to3}\dfrac{2x+10-16}{(x-3)(\sqrt{2x+10}+4)}=\lim\limits_{x\to3}\dfrac{2(x+3)}{(x-3)(\sqrt{2x+10}+4)}$ $\quad 2x+6$

$=\lim\limits_{x\to3}\dfrac{2}{\sqrt{2x+10}+4}=\dfrac{2}{4+4}=\dfrac{2}{8}=\dfrac{1}{4}$

22) $f(x)=\dfrac{\sqrt{4x^2+5}}{1-3x}$ $\quad -\infty$

$\lim\limits_{x\to-\infty} f(x)=\dfrac{\infty}{\infty}$ عدم تعيين

$f(x)=\dfrac{-x\sqrt{4+\frac{5}{x^2}}}{x(\frac{1}{x}-3)}$

$\lim\limits_{x\to-\infty}\dfrac{-2}{-3}=\dfrac{2}{3}$

23) $f(x)=\sqrt{x^2+1}-x$ $\quad +\infty$

$\lim\limits_{x\to+\infty} f(x)=\infty-\infty$ عدم تعيين

$\lim\limits_{x\to+\infty}\dfrac{1}{\sqrt{x^2+1}+x}=0$

24) $f(x)=2x+1-\sqrt{x^2+x-2}$ $\quad +\infty$

$\lim\limits_{x\to+\infty} f(x)=\infty-\infty$

$\lim\limits_{x\to+\infty} x\left[2+\frac{1}{x}-\sqrt{1+\frac{1}{x}-\frac{2}{x}}\right]$

$=+\infty[2-1]=+\infty$

25) $f(x)=x-\sqrt{x}$ $\quad +\infty$

$\lim\limits_{x\to+\infty} f(x)=+\infty$

26) $f(x)=\sqrt[3]{x^2+5x+3}$ $\quad 1$

$\lim\limits_{x\to1} f(x)=\sqrt[3]{9}$

27) $f(x)=\sqrt{\dfrac{x^2-2x+1}{1+2x}}$ $\quad 1$

$\lim\limits_{x\to1} f(x)=0$

28) $f(x)=2\sqrt{x}-\sqrt{4x+1}$ $\quad +\infty$

$\lim\limits_{x\to+\infty} f(x)=\infty-\infty$

$\lim\limits_{x\to\infty}\dfrac{4x-4x-1}{2\sqrt{x}+\sqrt{4x+1}}=0$

$$f(x)=\frac{-x(\sqrt{x+1}+1)}{x(\sqrt{4-x}+2)}$$

$$\lim_{x\to 0} f(x)=\frac{1+1}{2+2}=\frac{2}{4}=\frac{1}{2}$$

36) $f(x)=\dfrac{x^2-4}{|x-2|}$ — 2

$$f(x)=\begin{cases}\dfrac{x^2-4}{-(x-2)} & x<2\\ \dfrac{x^2-4}{x-2} & x>2\end{cases}$$

x	$-\infty$		2		$+\infty$
$x-2$		−	0	+	

$$f(x)=\begin{cases}\dfrac{(x-2)(x+2)}{-(x-2)} & x<2\\ \dfrac{(x-2)(x+2)}{x-2} & x>2\end{cases}$$

$$\lim_{x\to 2^-} f(x)=-4 \qquad \lim_{x\to 2^+} f(x)=4$$

37) $f(x)=\dfrac{\sqrt{x}-1}{x^2-1}$ — 1

$\lim_{x\to 1} f(x)=\frac{0}{0}$ عدم تعيين

$$f(x)=\frac{x-1}{(x^2-1)(\sqrt{x}+1)}$$

$$f(x)=\frac{x-1}{(x-1)(x+1)(\sqrt{x}+1)}$$

$$\lim_{x\to 1} f(x)=\frac{1}{4}$$

38) $f(x)=\dfrac{|x^3-4x|}{x+2}$ — -2

$x(x^2-4)$

	$-\infty$		-2		0		2		$+\infty$
x		−	!	−	0	+	!	+	
x^2-4		+	0	−	!	−	0	+	
جداء		−	0	+	0	−	0	+	

$$f=\begin{cases}\dfrac{x^3-4x}{x+2} & x>-2\\ \dfrac{-(x^3-4x)}{x+2} & x<-2\end{cases}$$

$$f(x)=\begin{cases}\dfrac{x(x-2)(x+2)}{x+2} & \lim\limits_{x\to -2^+} f(x)=8\\ \dfrac{-x(x-2)(x+2)}{x+2} & \lim\limits_{x\to -2^-} f(x)=-8\end{cases}$$

99) $f(x)=\dfrac{3x^5-2x+1}{x^3-1}$ — $-\infty$

$$\lim_{x\to -\infty} f(x)=\lim_{x\to -\infty}\left(\frac{3x^5}{x^3}\right)=+\infty$$

30) $f(x)=\sqrt{x}-2x+1$ — $+\infty$

$$\lim_{x\to +\infty} f(x)=-\infty$$

31) $f(x)=f(x)=9$ — 8

$\lim_{x\to 8} f(x)=9$

32) $f(x)=-5x$ — $-\infty$

$\lim_{x\to -\infty} f(x)=+\infty$

33) $f(x)=\dfrac{6-x^3}{7x^3+1}$ — $+\infty$

$\lim_{x\to +\infty} f(x)=\frac{-1}{7}$

34) $f(x)=\dfrac{x^3-1}{x^2-1}$ — 1 , −1

$\lim_{x\to 1} f(x)=\frac{0}{0}$ عدم تعيين

$$\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}=\frac{3}{2}$$

35) $f(x)=\dfrac{\sqrt{4-x}-2}{\sqrt{x+1}-1}$ — 0

$\lim_{x\to 0} f(x)=0$

نضرب بمرافق البسط

$$f(x)=\frac{-x-x-x}{(\sqrt{x+1}-1)(\sqrt{4-x}+2)}$$

نضرب بمرافق المقام

$$f(x)=\frac{-x(\sqrt{x+1}+1)}{(x+1-1)(\sqrt{4-x}+2)}$$

$$f(x)=\frac{\sqrt{2}\,\frac{1}{\sqrt{2}}(\cos x-\sin x)}{x-\frac{\pi}{4}}$$

$$=\sqrt{2}\,\frac{\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x}{x-\frac{\pi}{4}}$$

$$=\sqrt{2}\,\frac{\sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x}{x-\frac{\pi}{4}}$$

$$=\sqrt{2}\,\frac{\sin(\frac{\pi}{4}-x)}{x-\frac{\pi}{4}}$$

$\theta=\frac{\pi}{4}-x$, $x\to\frac{\pi}{4}$, $\theta\to 0$

$$=\sqrt{2}\,\frac{\sin(\frac{\pi}{4}-x)}{-(\frac{\pi}{4}-x)}=-\sqrt{2}$$

45) $f=\dfrac{x-2x^2+x^3}{\cos(2x-2)-1}$ — 1

$$f=\frac{x(x^2-2x+1)}{-(1-\cos 2(x-1))}$$

$$=\frac{x(x-1)^2}{-2\sin^2(x-1)}$$

$\theta=x-1$, $x\to 1$, $\theta\to 0$

$$=\frac{x}{-2}\left(\frac{x-1}{\sin(x-1)}\right)^2=\frac{-1}{2}$$

46) $f(x)=\dfrac{\sin(x-1)}{x^2-1}$ — 1

$$f(x)=\frac{\sin(x-1)}{(x-1)(x+1)}$$

$$=\frac{\sin(x-1)}{x-1}\cdot\frac{1}{x+1}=\frac{1}{2}$$

47) $f(x)=\dfrac{\sqrt{2x^3-1}-1}{x^2-1}$ — 1

$$=\frac{2x^3-1-1}{(x^2-1)(\sqrt{2x^3-1}+1)}=\frac{2x^3-2}{(x^2-1)(\sqrt{2x^3-1}+1)}$$

39) $f(x)=\dfrac{\sqrt{x^2-1}}{\sqrt{x^2+1}}$ — $-\infty$

$\lim_{x\to-\infty} f(x)=1$

40) $f(x)=\dfrac{\sin^2 x}{3x^2}$ — 0

$$\lim_{x\to 0}\frac{1}{3}\left(\frac{\sin x}{x}\right)^2=\frac{1}{3}$$

41) $f(x)=\dfrac{\cos x-1}{x}$ — 0

$$\lim_{x\to 0}\frac{-(1-\cos x)}{x}=\lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{x}$$

$$=\lim_{x\to 0}-2\,\frac{\sin\frac{x}{2}\sin\frac{x}{2}}{2\cdot\frac{x}{2}}=0$$

42) $f(x)=\dfrac{\cos x-1}{\sin x}$ — 0

$\lim_{x\to 0} f(x)=\frac{0}{0}$ حالة تعيين

$$f(x)=\frac{-(1-\cos x)}{\sin x}=\frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$f(x)=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\qquad \lim_{x\to 0} f(x)=\frac{0}{1}=0$$

43) $f(x)=\dfrac{\sqrt{x+1}-1}{\tan x}$ — 0

نضرب بالمرافق

$$f(x)=\frac{x+1-1}{\tan x(\sqrt{x+1}+1)}$$

$$=\frac{x}{\tan x(\sqrt{x+1}+1)}$$

$\lim_{x\to 0} f=\frac{1}{2}$

44) $f(x)=\dfrac{\cos x-\sin x}{x-\frac{\pi}{4}}$ — $\frac{\pi}{4}$

51) $f(x)=\dfrac{\sin 2x}{\sqrt{2x+3}-\sqrt{3}}$ ... 0

$\lim_{x\to 0} f(x)=\frac{0}{0}$ حالة عدم تعيين

$f(x)=\dfrac{\sin 2x\,(\sqrt{2x+3}+\sqrt{3})}{2x+3-3}$

$f(x)=\dfrac{\sin 2x}{2x}(\sqrt{2x+3}+\sqrt{3})$

$\lim_{x\to 0} f(x)=1\,(\sqrt{3}+\sqrt{3})=2\sqrt{3}$

52) $f(x)=\dfrac{\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}}{x}$ ... 0

$\lim_{x\to 0} f(x)=\frac{0}{0}$ حالة عدم تعيين

$f(x)=\dfrac{1+\sin 2x-1+\sin 2x}{x(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x})}$

$=\dfrac{2\,2\sin 2x}{2\cdot x(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x})}$

$=\dfrac{4\cdot \sin 2x}{2x(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x})}$

$\lim_{x\to 0} f(x)=4(1)\dfrac{1}{1+1}=2$

53) $f(x)=\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$ ... 0

$f(x)=\dfrac{1+x^2-\cos^2 x}{x^2(\sqrt{1+x^2}+\cos x)}$

$1-\cos^2 x=\sin^2 x$

$=\dfrac{x^2+\sin^2 x}{x^2(\sqrt{1+x^2}+\cos x)}$

$=\dfrac{x^2\left(1+\frac{\sin^2 x}{x^2}\right)}{x^2(\sqrt{1+x^2}+\cos x)}$

$\lim_{x\to 0} f=\dfrac{1+1}{1+1}=1$

$f(x)=\dfrac{2(x-1)(x^2+x+1)}{(x-1)(x+1)(\sqrt{2x^3-1}+1)}$

$\lim_{x\to 1} f(x)=\dfrac{2(3)}{2(2)}=\dfrac{3}{2}$

48) $f(x)=\dfrac{x\cdot\cos x}{x^2+1}$ ... $+\infty$

$-1\le\cos x\le 1$

نضرب بـ $\dfrac{x}{x^2+1}>0$

$\dfrac{-x}{x^2+1}\le\dfrac{x\cos x}{x^2+1}\le\dfrac{x}{x^2+1}$

$\lim_{x\to+\infty}\left(\frac{x}{x^2+1}\right)$, $\lim_{x\to+\infty}\left(\frac{-x}{x^2+1}\right)$ $\Rightarrow \lim_{x\to+\infty} f(x)=0$ حسب الحصر

49) $f(x)=\dfrac{\tan 3x-x}{x+2\sin x}$ ... 0

نقسم البسط والمقام على $x>0$

$f(x)=\dfrac{3\frac{\tan 3x}{3x}-1}{1+2\frac{\sin x}{x}}$

$\lim_{x\to 0} f(x)=\dfrac{3-1}{1+2}=\dfrac{2}{3}$

50) $f(x)=\dfrac{\sin 2x}{\sqrt{x+1}-1}$ ... 0

$f(x)=\dfrac{\sin 2x\,(\sqrt{x+1}+1)}{x+1-1}$

$=\dfrac{2\sin 2x\,(\sqrt{x+1}+1)}{2x}$

$\lim_{x\to 0} f(x)=2(1+1)=4$

$$= \frac{-2\sin^2\frac{1}{2}(\frac{\pi}{2}-x)}{-2(\frac{\pi}{2}-x)}$$

$$\lim_{x\to\frac{\pi}{2}} f(x) = 0$$

59) $f(x) = \frac{\sqrt{x+9}-3}{\sin 7x}$ $\quad 0$

$$= \frac{x+9-9}{\sin 7x(\sqrt{x+9}+3)}$$

$$= \frac{7x}{7\sin 7x(\sqrt{x+9}+3)}$$

$$\lim_{x\to 0} f(x) = \frac{1}{7(6)} = \frac{1}{42}$$

60) $f(x) = x\sin\frac{1}{x}$ $\quad +\infty$

$$f(x) = \frac{\sin\frac{1}{x}}{\frac{1}{x}} \qquad \theta = \frac{1}{x}, \ x\to+\infty, \ \theta\to 0$$

$$\lim_{x\to+\infty} f(x) = 1$$

61) $f(x) = \frac{\cos x}{x-\frac{\pi}{2}}$ $\quad \frac{\pi}{2}$

$$f(x) = \frac{\sin(\frac{\pi}{2}-x)}{-(\frac{\pi}{2}-x)}$$

$$\lim_{x\to\frac{\pi}{2}} f(x) = -1$$

62) $f(x) = \frac{1-\sqrt{\cos x}}{\tan^2 x}$ $\quad 0$

$$= \frac{(1-\cos x)(1+\cos x)}{\tan^2 x(1+\sqrt{\cos x})}$$

$$= \frac{1-\cos^2 x}{\tan^2 x(1+\sqrt{\cos x})}$$

54) يوجد طريقة ثانية

55) $f(x) = \frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$ نضرب بالمرافق

$$f(x) = \frac{1+\sin^2 x-\cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x+1-\cos^2 x}{\sin^2 x(\sqrt{\sin^2 x+1}+\cos x)}$$

$$= \frac{\sin^2 x+\sin^2 x = 2\sin^2 x}{\sin^2 x(\sqrt{1+\sin^2 x}+\cos x)}$$

$$\lim_{x\to 0} f(x) = \frac{2}{1+1} = 1$$

56) $f(x) = \frac{1-\cos x+\sin x}{x}$ $\quad 0$

$$f(x) = \frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{x}$$

$$= \frac{2\sin\frac{x}{2}(\sin\frac{x}{2}+\cos\frac{x}{2})}{2\frac{x}{2}}$$

$$\lim_{x\to 0} f(x) = 1(0+1) = 1$$

57) $f(x) = \frac{1}{\sqrt{x}}\sin x$ $\quad 0$

$$f(x) = \sqrt{x}\cdot\frac{\sin x}{\sqrt{x}\sqrt{x}} = \sqrt{x}\,\frac{\sin x}{x}$$

$$\lim_{x\to 0} f(x) = 0(1) = 0$$

58) $f(x) = \frac{\sin x-1}{x-\frac{\pi}{2}}$ $\quad \frac{\pi}{2}$

$$= \frac{\cos(\frac{\pi}{2}-x)-1}{x-\frac{\pi}{2}} = \frac{-(1-\cos(\frac{\pi}{2}-x))}{x-\frac{\pi}{2}}$$

$$= \frac{-2\sin^2\frac{1}{2}(\frac{\pi}{2}-x)}{x-\frac{\pi}{2}}$$

67) $f(x) = \dfrac{\sin x - 2x}{x - 2\sin x}$

$= \dfrac{x\left(\frac{\sin x}{x} - 2\right)}{x\left(1 - 2\frac{\sin x}{x}\right)}$ $\lim = \dfrac{1-2}{1-2} = 1$

68) $f(x) = \dfrac{4x}{\sin 2x}$ $\quad 0$

$= \dfrac{2 \cdot 2x}{\sin 2x}$ $\lim_{x \to 0} f(x) = 2(1) = 2$

69) $f(x) = \dfrac{\sin(\sin 2x)}{\sin 5x}$ $\quad 0$

$f(x) = \dfrac{\sin(\sin 2x)}{\sin 2x} \times \dfrac{\sin 2x}{\sin 5x}$

$= \dfrac{\sin(\sin 2x)}{\sin 2x} \times \dfrac{2\frac{\sin 2x}{2x}}{5\frac{\sin 5x}{5x}}$

$\theta = \sin 2x$, $x \to 0$, $\theta \to 0$ $\lim_{x \to 0} f = 1 \times \frac{2}{5} = \frac{2}{5}$

70) $f = \dfrac{\sin x + \sin 2x + \sin 3x}{\sin 4x + \sin 5x + \sin 6x}$

$= \dfrac{x\left(\frac{\sin x}{x} + 2\frac{\sin 2x}{2x} + 3\frac{\sin 3x}{3x}\right)}{x\left(4\frac{\sin 4x}{4x} + 5\frac{\sin 5x}{5x} + 6\frac{\sin 6x}{6x}\right)}$

$\lim = \dfrac{1+2+3}{4+5+6} = \dfrac{6}{15}$

71) $f = \dfrac{\sin x - \sin 3x}{x}$

$= \dfrac{\sin x}{x} - 3\dfrac{\sin 3x}{3x}$ $\lim = 1 - 3 = -2$

$= \dfrac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}(1 + \sqrt{\cos x})}$

$= \dfrac{\cos^2 x}{1 + \sqrt{\cos x}}$

$\lim_{x \to 0} f(x) = \dfrac{1}{1+1} = \dfrac{1}{2}$

63) $f(x) = \dfrac{x + \tan x}{x + \sin x}$

$= \dfrac{x\left(1 + \frac{\tan x}{x}\right)}{x\left(1 + \frac{\sin x}{x}\right)}$

$\lim_{x \to 0} f(x) = 1$

64) $f(x) = \dfrac{x \cdot \sin x}{\sin^2 5x}$ (تقسيم بسطاً ومقاماً على x^2 لأن $x^2 > 0$)

$= \dfrac{\frac{x \cdot \sin x}{x^2}}{\frac{\sin^2 5x}{x^2}}$

$= \dfrac{\frac{\sin x}{x}}{\left(5\frac{\sin 5x}{5x}\right)^2}$

$\lim_{x \to 0} f(x) = \dfrac{1}{25}$

65) $f(x) = \dfrac{x^2}{\sin^2 3x}$ $\quad 0$

$= \left(\dfrac{x}{\sin 3x}\right)^2 = \left(\dfrac{3x}{3\sin 3x}\right)^2$

$\lim_{x \to 0} f(x) = \dfrac{1}{9}$

66) $f(x) = \dfrac{\sin 3x}{\sin 4x}$ $\quad 0$

$= \dfrac{3\frac{\sin 3x}{3x}}{4\frac{\sin 4x}{4x}}$ $\lim = \dfrac{3}{4}$

72) $f(x)=\dfrac{x+\sin x}{2x-3\sin 2x}$ ∞

$f(x)=\dfrac{x\left(1+\frac{\sin x}{x}\right)}{x\left(2-3\frac{\sin 2x}{x}\right)}$

$=\dfrac{1+\frac{\sin x}{x}}{2-6\frac{\sin 2x}{2x}} \Rightarrow \ell=\dfrac{1+1}{2-6}=\dfrac{2}{-4}$

$=-\dfrac{1}{2}$

73) $f(x)=\dfrac{\tan(mx)}{\sin(nx)}$ ∞

$=\dfrac{m\frac{\tan(mx)}{mx}}{n\frac{\sin(nx)}{nx}} \quad \ell=\dfrac{m}{n}$

74) $f(x)=\dfrac{x\cdot\sin x}{1-\cos^2 2x}$ ∞

$=\dfrac{x\cdot\sin x}{\sin^2 x} \quad =\dfrac{x\cdot\sin x}{4\sin^2 x\cdot\cos^2 x}$

$=\dfrac{x}{4\sin x\cos^2 x} \quad \ell=\dfrac{1}{4}$

75) $f(x)=\dfrac{\sin^2 x-x^2}{3x^2}$ ∞

$=\dfrac{1}{3}\dfrac{\sin^2 x}{x^2}-\dfrac{1}{3}\dfrac{x^2}{x^2}$

$=\dfrac{1}{3}\left(\dfrac{\sin x}{x}\right)^2-\dfrac{1}{3}$

$\lim_{x\to 0} f(x)=\dfrac{1}{3}-\dfrac{1}{3}=0$

76) $f(x)=\dfrac{3\sin 2x}{5x}$ ∞

$P=\dfrac{3}{5}\dfrac{\sin 2x}{2x}\,2$

$\ell=\dfrac{6}{5}$

78) $f(x)=\dfrac{\sin(2025x)}{\sin(2026x)}$

$=\dfrac{2025\frac{\sin(2025x)}{2025x}}{2026\frac{\sin(2026x)}{2026x}}$

$\ell=\dfrac{2025}{2026}$

79) $f(x)=\cos\left(\pi\dfrac{\sin x}{x}\right)$ 0

$\lim_{x\to 0} f(x)=\cos(\pi(1))=-1$

80) $f(x)=\dfrac{\sqrt{1-\cos x}}{\tan x}$ 0

$=\dfrac{\sqrt{2\sin^2\frac{x}{2}}}{\tan x}=\dfrac{\sqrt{2}\sin\frac{x}{2}}{\frac{\sin x}{\cos x}}$

$=\dfrac{\sqrt{2}\sin\frac{x}{2}\cdot\cos x}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}} \quad \ell=\dfrac{\sqrt{2}}{2}$

81) $f(x)=\dfrac{\sin(x^2-x)}{x-1}$ 1

$=x\dfrac{\sin(x(x-1))}{x(x-1)} \qquad \theta=x(x-1)$

$x\to 1,\ \theta\to 0$

$\ell=1(1)=1$

82) $P=\dfrac{\sin x-\cos x}{x-\frac{\pi}{4}}$ $\frac{\pi}{4}$

$=\sqrt{2}\dfrac{\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x}{x-\frac{\pi}{4}}$

$$f(x)=\sqrt{2}\,\frac{\sin x\cdot\cos\left(\frac{\pi}{4}\right)-\sin\frac{\pi}{4}\cos x}{x-\frac{\pi}{4}}$$

$$=\sqrt{2}\,\frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$$

$$\lim_{x\to\frac{\pi}{4}} f=\sqrt{2}$$

83) $f(x)=\dfrac{\tan x-\sin x}{x^3}$ عند 0 نوحد ونضرب

$$=\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}$$

$$=\frac{\sin x-\cos x\cdot\sin x}{\cos x\cdot x^3}$$

$$=\frac{\sin x(1-\cos x)}{x\cdot x^2}\cdot\frac{1}{\cos x}$$

$$=\frac{\sin x}{x}\cdot\frac{2\sin^2}{x^2}\cdot\frac{1}{\cos x}$$

$$=\frac{\sin x}{x}\cdot 2\left(\frac{\sin\ldots}{2\frac{x}{2}}\right)^2\cdot\frac{1}{\cos x}$$

$$\lim_{x\to 0} f(x)=1\cdot 2\left(\frac{1}{2}\right)^2\cdot 1=\frac{1}{2}$$

84) $f(x)=\dfrac{\sin(\pi x)}{x-1}$ عند 1

نفرض $x-1=t\Rightarrow x=t+1$ ، $x\to 1$ ، $t\to 0$

$$f(x)=\frac{\sin(\pi+\pi t)}{t}=\frac{-\sin\pi t}{t}$$

$$=-\pi\frac{\sin\pi t}{\pi t}$$

$$\lim_{x\to 1} f=-\pi$$

85) $f(x)=f(x)=\dfrac{\cos x}{\cos(2x)}$ عند $\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\frac{\cos\frac{\pi}{2}}{\cos\pi}=\frac{0}{-1}=0$$

86) $f(x)=\dfrac{\sin 2x-2\sin x}{x^3}$

$$=\frac{2\sin x\cdot\cos x-2\sin x}{x^3}$$

$$=\frac{-2\sin x(1-\cos x)}{x\cdot x^2}$$

$$=\frac{-2\sin x}{x}\cdot\frac{2\sin^2\frac{x}{2}}{x^2}$$

$$=-2\frac{\sin x}{x}\cdot 2\left(\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\right)^2$$

$$\lim_{x\to 0} f=-2(1)\cdot 2\left(\frac{1}{2}\right)^2=-1$$

87) $f(x)=\dfrac{2\sin x-1}{6x-\pi}$ عند $\frac{\pi}{6}$

$$f(x)=\frac{2\left(\sin x-\frac{1}{2}\right)}{6x-\pi}$$

$$=\frac{2\left(\sin x-\sin\frac{\pi}{6}\right)}{6x-\pi}$$

$$=\frac{2\cdot 2\cos\left(\frac{x+\frac{\pi}{6}}{2}\right)\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)}{6x-\pi}$$

$$=\frac{4\cos\left(\frac{6x+\pi}{12}\right)\sin\left(\frac{6x-\pi}{12}\right)}{12\,\frac{6x-\pi}{12}}$$

$$\lim=\frac{4}{12}\cos\left(\frac{\pi}{6}\right)=\frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{6}$$

دساتير التحويل سنجرع فيها

$$\sin a+\sin b=2\sin\left(\frac{a+b}{2}\right)\cdot\cos\left(\frac{a-b}{2}\right)$$

$$\sin a-\sin b=2\cos\left(\frac{a+b}{2}\right)\cdot\sin\left(\frac{a-b}{2}\right)$$

$$\cos a+\cos b=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos a-\cos b=-2\sin\left(\frac{a+b}{2}\right)\cdot\sin\left(\frac{a-b}{2}\right)$$

$$= \frac{\cos x \sqrt[3]{1+\sin x}}{\sqrt{(1-\sin x)(1+\sin x)}}$$

$$= \frac{\cos x \cdot \sqrt[3]{1+\sin x}}{\sqrt[3]{1-\sin^2 x}} = \frac{\cos x \sqrt[3]{1+\sin x}}{\sqrt[3]{\cos^2 x}}$$

$$= \frac{\cos x \cdot \sqrt[3]{1+\sin x}}{\cos x^{\frac{2}{3}}} = \cos x^{-\frac{2}{3}} \cos x \sqrt[3]{1+\sin x}$$

$$= \cos x^{\frac{1}{3}} \cdot \sqrt[3]{1+\sin x}$$

$$= \sqrt[3]{\cos x (1+\sin x)}$$

$$\lim f = \sqrt[3]{0(1+1)} = 0$$

92) $f = \dfrac{x^2 - 1 + \cos^2 x}{x \cdot \sin x}$ 0

$$f(x) = \frac{x^2 - (1-\cos^2 x)}{x \cdot \sin x}$$

$$= \frac{x^2 - \sin^2 x}{x \cdot \sin x} = \frac{1 - \frac{\sin^2 x}{x^2}}{\frac{x \cdot \sin x}{x^2}}$$

$$\lim_{x \to 0} f(x) = \frac{1-1}{1} = 0$$

93) $f(x) = \cos\left(\frac{\pi x - 1}{x - 2}\right)$ $+\infty$

$$\lim_{x \to +\infty} f(x) = \cos(\pi) = -1$$

94) $f(x) = \sin\left(\frac{\pi x + 1}{2x + 3}\right)$ $+\infty$

$$\lim_{x \to +\infty} f(x) = \sin\left(\frac{\pi}{2}\right) = 1$$

88) $f(x) = \dfrac{\sin(2x)}{\sqrt{1-\cos x}}$ $0+$

$$= \frac{\sin 2x}{\sqrt{2\sin^2 \frac{x}{2}}} = \frac{\sin 2x}{\sqrt{2}\sin\frac{x}{2}}$$

$$= \frac{2\frac{\sin 2x}{2x}}{\sqrt{2}\frac{\sin\frac{x}{2}}{2\frac{x}{2}}} \Rightarrow \lim = \frac{\sqrt{2}(1)}{\frac{\sqrt{2}}{2}} = 2$$

89) $f(x) = \dfrac{\sin 2x}{\sqrt{1+x} - 1}$ 0

$$f(x) = \frac{\sin 2x (\sqrt{1+x} + 1)}{x + x - x}$$

$$= 2\frac{\sin 2x (\sqrt{1+x} + 1)}{2x}$$

$$\lim = 2(1)(1+1) = 4$$

90) $f(x) = \dfrac{\sin^2 9x - x^2}{1 - \cos 2x}$ 0

$$= \frac{\sin^2 9x - x^2}{2\sin^2 x} = \frac{\left(\frac{\sin^2 9x}{x^2} - 1\right)}{2\frac{\sin^2 x}{x^2}}$$

$$= \frac{\left(9\frac{\sin 9x}{9x}\right)^2 - 1}{2\left(\frac{\sin x}{x}\right)^2} \quad \lim = \frac{81 - 1}{2} = 40$$

91) $f(x) = \dfrac{\cos x}{\sqrt[3]{1-\sin x}}$ $\frac{\pi}{2}$

$$= \frac{\cos x \sqrt[3]{1+\sin x}}{\sqrt[3]{1-\sin x} \cdot \sqrt[3]{1+\sin x}}$$

95) $f=\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ ○

$f=\dfrac{1+\sin x-1+\sin x}{\tan x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$

$=\dfrac{2\sin x}{\frac{\sin x}{\cos x}\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$

$=\dfrac{2\cos x}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

$\lim\limits_{x\to 0} f(x)=\dfrac{2}{1+1}=1$

96) $f(x)=\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$ ○

نضرب بالمرافق.

$=\dfrac{1+x^2-\cos^2 x}{x^2\left(\sqrt{1+x^2}+\cos x\right)}$

$=\dfrac{x^2+\sin^2 x}{x^2\left(\sqrt{1+x^2}+\cos x\right)}$ نقسم بسط ومقام على $x^2\neq 0$

$f(x)=\dfrac{1+\frac{\sin^2 x}{x^2}}{\sqrt{1+x^2}+\cos x}$

$\lim\limits_{x\to 0} f(x)=\dfrac{1+1}{1+1}=1$

97) $f(x)=\dfrac{1-\cos 4x}{x^2}$ ○

$=\dfrac{2\sin^2 2x}{x^2}=2\left(2\dfrac{\sin 2x}{2x}\right)^2$

$\lim\limits_{x\to 0} f(x)=2(2)^2=8$

98) $f(x)=\dfrac{x\cdot\sin 3x}{1-\cos x}$ ○

$=\dfrac{x\cdot\sin 3x}{2\sin^2\frac{x}{2}}=\dfrac{\frac{x\cdot\sin 3x}{x^2}}{\frac{2\sin^2\frac{x}{2}}{x^2}}$

$=\dfrac{3\frac{\sin 3x}{3x}}{2\left(\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\right)^2}$ $\lim=\dfrac{3}{\frac{1}{2}}=6$

99) $f(x)=\dfrac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$ ○

$=\dfrac{1+\sin^2 x-\cos^2 x}{\sin^2 x\left(\sqrt{1+\sin^2 x}+\cos x\right)}$

$=\dfrac{\sin^2 x+\sin^2 x}{\sin^2 x\left(\sqrt{1+\sin^2 x}+\cos x\right)}$

$=\dfrac{2}{\sqrt{1+\sin^2 x}+\cos x}$ $\lim=\dfrac{2}{1+1}=1$

100) $f(x)=\dfrac{(a-b)x}{\sin ax-\sin bx}$ ○

$=\dfrac{a-b}{a\frac{\sin ax}{ax}-b\frac{\sin bx}{bx}}$

$\lim f=\dfrac{a-b}{a(1)-b(1)}=\dfrac{a-b}{a-b}=1$

انتهى الحل بحمد الله وتوفيقه