**حل كلاً من النهايات الآتية :** ***Solve all of these limits :***

1) $\lim_{x\to 0}\frac{\sin 2x}{\sqrt{x+1}-1}$

2) $\lim_{x\to 0}\frac{\sin 2x}{\sqrt{2x+3}-\sqrt{3}}$

3) $\lim_{x\to 0}\frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$

4) $\lim_{x\to 0}\frac{\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}}{x}$

5) $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\cos x}{x^2}$

6) $\lim_{x\to 0}\frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$

7) $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

8) $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{\sin 7x}$

9) $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

10) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

11) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

12) $\lim_{x\to 0}\frac{1-\cos 3x}{x^2}$

13) $\lim_{x\to 0}\frac{1-\cos 4x}{x^2}$

14) $\lim_{x\to 0}\frac{1-\cos ax}{x^2}$

15) $\lim_{x\to 0}\frac{x\sin 3x}{1-\cos x}$

16) $\lim_{x\to 0}\frac{1-\cos x+\sin x}{x}$

17) $\lim_{x\to 0}\frac{1-\cos 3x}{1-\cos 5x}$

18) $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

19) $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\tan^2 x}$

20) $\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{\sin 2x}$

21) $\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}$

22) $\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{\tan^2 x}$

23) $\lim_{x\to 0}\frac{1+x-\cos 2x}{1-\sqrt{1+\sin x}}$

24) $\lim_{x\to 0}\frac{(a-b)x}{\sin ax-\sin bx}$

25) $\lim_{x\to 0}\frac{1-\cos x\cos 2x}{x^2}$

26) $\lim_{x\to 0}\frac{\cos^2 x-\cos x}{x^2}$

27) $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

28) $\lim_{x\to 0}\frac{x\sin 2x}{1-\cos x}$

29) $\lim_{x\to 0}\frac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}$

30) $\lim_{x\to 0}\frac{x}{x+\sin x}$

31) $\lim_{x\to 0}\frac{\sin x}{\sqrt{x}}$

32) $\lim_{x\to 0}\frac{\sin^2 9x-x^2}{1-\cos 2x}$

33) $\lim_{x\to 0}\frac{1-\cos x}{\sin x}$

34) $\lim_{x\to -\frac{\pi}{4}}\frac{\sin x+\cos x}{x+\frac{\pi}{4}}$

35) $\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\sqrt[3]{1-\sin x}}$

36) $\lim_{x\to 0}\frac{x^2-1+\cos^2 x}{x\sin x}$

37) $\lim_{x\to \frac{\pi}{2}}\frac{\sin x-1}{x-\frac{\pi}{2}}$

**-انتهت الأسئلة-**

## حل ورقة عمل النهايات رقم ١

ملاحظة: في كل من النهايات الآتية نكتب $\frac{0}{0}$ حالة عدم تعيين نزيلها .
المكتوب باللون الأخضر يدل على المرافق
يمكن الحل بطرق أخرى (تتعدد الطرق و الجواب واحد)

1) $$\lim_{x\to 0}\frac{\sin 2x}{\sqrt{x+1}-1}=\lim_{x\to 0}\frac{\sin 2x}{x}\left(\sqrt{x+1}+1\right)$$
$$=2\lim_{x\to 0}\frac{\sin 2x}{2x}\left(\sqrt{x+1}+1\right)=(2)(1)(2)=4$$

2) $$\lim_{x\to 0}\frac{\sin 2x}{\sqrt{2x+3}-\sqrt{3}}=\lim_{x\to 0}\frac{\sin 2x}{2x}\left(\sqrt{2x+3}+\sqrt{3}\right)$$
$$=(1)\left(2\sqrt{3}\right)=2\sqrt{3}$$

3) $$\lim_{x\to 0}\frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}=\lim_{x\to 0}\frac{\sin 3x}{x}\left(\sqrt{x+2}+\sqrt{2}\right)$$
$$=3\lim_{x\to 0}\frac{\sin 3x}{3x}\left(\sqrt{x+2}+\sqrt{2}\right)=(3)(1)\left(2\sqrt{2}\right)=6\sqrt{2}$$

4) $$\lim_{x\to 0}\frac{\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}}{x}$$
$$=\lim_{x\to 0}\frac{1+\sin 2x-1+\sin 2x}{x\left(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x}\right)}$$
$$=\lim_{x\to 0}\frac{2\sin 2x}{x\left(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x}\right)}$$
$$=\lim_{x\to 0}\frac{4\sin 2x}{2x}\frac{1}{\left(\sqrt{1+\sin 2x}+\sqrt{1-\sin 2x}\right)}$$
$$=(4)(1)\left(\frac{1}{2}\right)=2$$

5) $$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\cos x}{x^2}=\lim_{x\to 0}\frac{1+x^2-\cos^2 x}{x^2\left(\sqrt{1+x^2}+\cos x\right)}$$
$$=\lim_{x\to 0}\frac{x^2+\sin^2 x}{x^2}\frac{1}{\sqrt{1+x^2}+\cos x}$$
$$=\lim_{x\to 0}\left(1+\left(\frac{\sin x}{x}\right)^2\right)\frac{1}{\sqrt{1+x^2}+\cos x}$$
$$=(1+(1)^2)\frac{1}{\sqrt{1}+1}=\frac{2}{2}=1$$

6) $$\lim_{x\to 0}\frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$$
$$=\lim_{x\to 0}\frac{1+\sin^2 x-\cos^2 x}{\sin^2 x\left(\sqrt{1+\sin^2 x}+\cos x\right)}$$

$1-\cos^2 x=\sin^2 x$

$$\lim_{x\to 0}\frac{2\sin^2 x}{\sin^2 x\left(\sqrt{1+\sin^2 x}+\cos x\right)}$$
$$=\lim_{x\to 0}\frac{2}{\left(\sqrt{1+\sin^2 x}+\cos x\right)}=\frac{2}{2}=1$$

7) $$\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$$
$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{1+x\sin x-\cos^2 x}$$
$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{\sin^2 x+x\sin x}$$

**نقسم البسط و المقام على $x^2$ :**

$$=\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2\left(\sqrt{1+x\sin x}+\cos x\right)}{\left(\frac{\sin x}{x}\right)^2+\frac{\sin x}{x}}$$
$$=\frac{(1)^2(\sqrt{1}+1)}{(1)^2+1}=\frac{2}{2}=1$$

8) $$\lim_{x\to 0}\frac{\sqrt{x+9}-3}{\sin 7x}=\lim_{x\to 0}\frac{x}{\sin 7x}\cdot\frac{1}{\sqrt{x+9}+3}$$
$$\lim_{x\to 0}\left(\frac{7x}{\sin 7x}\right)\cdot\left(\frac{1}{7}\right)\cdot\left(\frac{1}{\sqrt{x+9}+3}\right)$$
$$(1)\cdot\left(\frac{1}{7}\right)\cdot\left(\frac{1}{6}\right)=\frac{1}{42}$$

9) $$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$$
$$=\lim_{x\to 0}\frac{2\sin x}{\tan x}\frac{1}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$
$$=\lim_{x\to 0}\frac{2\cos x}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}=\frac{2}{2}=1$$

10) $$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\left(\frac{1+\cos x}{1+\cos x}\right)$$
$$=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}$$
$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\frac{1}{1+\cos x}=(1)^2\frac{1}{1+1}=\frac{1}{2}$$

$$\lim_{x\to 0}\left(\frac{2\sin^2\frac{x}{2}}{x}+\frac{\sin x}{x}\right)=\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\sin\frac{x}{2}+\frac{\sin x}{x}\right)$$
$$=(1)(0)+1=1$$

17) $\lim_{x\to 0}\frac{1-\cos 3x}{1-\cos 5x}\left(\frac{1+\cos 3x}{1+\cos 5x}\right)\left(\frac{1+\cos 5x}{1+\cos 3x}\right)$

$$=\lim_{x\to 0}\frac{1-\cos^2 3x}{1-\cos^2 5x}\left(\frac{1+\cos 5x}{1+\cos 3x}\right)$$
$$=\lim_{x\to 0}\frac{\sin^2 3x}{\sin^2 5x}\left(\frac{1+\cos 5x}{1+\cos 3x}\right)$$
$$=\lim_{x\to 0}\left(\frac{\sin 3x}{\sin 5x}\right)^2\left(\frac{1+\cos 5x}{1+\cos 3x}\right)$$

$$=\lim_{x\to 0}\left(\frac{3}{5}\frac{\frac{\sin 3x}{3x}}{\frac{\sin 5x}{5x}}\right)^2\left(\frac{1+\cos 5x}{1+\cos 3x}\right)=\left(\frac{3}{5}(1)\right)^2\frac{2}{2}=\frac{9}{25}$$

18) $\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

$$1-\cos x=2\sin^2\frac{x}{2}$$

$$\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}+\sin x}{2\sin^2\frac{x}{2}-\sin x}$$

نقسم البسط و المقام على $x$ :

$$\lim_{x\to 0}\frac{2\frac{\sin^2\frac{x}{2}}{x}+\frac{\sin x}{x}}{2\frac{\sin^2\frac{x}{2}}{x}-\frac{\sin x}{x}}$$

$$=\lim_{x\to 0}\frac{\frac{\sin\frac{x}{2}}{\frac{x}{2}}\sin\frac{x}{2}+\frac{\sin x}{x}}{\frac{\sin\frac{x}{2}}{\frac{x}{2}}\sin\frac{x}{2}-\frac{\sin x}{x}}=\frac{(1)(0)+1}{(1)(0)-1}=-1$$

19) $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\tan^2 x}$

$$=\lim_{x\to 0}\frac{2-1-\cos x}{\tan^2 x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{\tan^2 x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

11) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}=2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2=2$

12) $\lim_{x\to 0}\frac{1-\cos 3x}{x^2}=\lim_{x\to 0}\frac{2\sin^2\frac{3x}{2}}{x^2}$

$$=\lim_{x\to 0}2\left(\frac{\sin\frac{3x}{2}}{x}\right)^2=\lim_{x\to 0}2\left(\frac{3}{2}.\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2$$
$$=2\left(\frac{3}{2}.1\right)^2=\frac{9}{2}$$

13) $\lim_{x\to 0}\frac{1-\cos 4x}{x^2}=\lim_{x\to 0}\frac{2\sin^2 2x}{x^2}$

$$\sin 2x=2\sin x\cos x$$

$$\lim_{x\to 0}\frac{1-\cos 4x}{x^2}=\lim_{x\to 0}\frac{2(2\sin x\cos x)^2}{x^2}$$
$$=\lim_{x\to 0}\frac{8\sin^2 x\cos^2 x}{x^2}=\lim_{x\to 0}8\left(\frac{\sin x}{x}\right)^2\cos^2 x$$
$$=(8)(1)(1)=8$$

14) $\lim_{x\to 0}\frac{1-\cos ax}{x^2}=\lim_{x\to 0}\frac{1-\cos ax}{x^2}\left(\frac{1+\cos ax}{1+\cos ax}\right)$

$$\lim_{x\to 0}\frac{1-\cos^2 ax}{x^2(1+\cos ax)}=\lim_{x\to 0}\frac{\sin^2 ax}{x^2(1+\cos ax)}$$

$$\lim_{x\to 0}\left(a\frac{\sin ax}{ax}\right)^2\frac{1}{1+\cos x}=\frac{a^2}{2}$$

15) $\lim_{x\to 0}\frac{x\sin 3x}{1-\cos x}=\lim_{x\to 0}\frac{x\sin 3x}{1-\cos x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$$=\lim_{x\to 0}\frac{x\sin 3x}{1-\cos^2 x}(1+\cos x)$$
$$=\lim_{x\to 0}\frac{x\sin 3x}{\sin^2 x}(1+\cos x)$$
$$=\lim_{x\to 0}\frac{x}{\sin x}\frac{\sin 3x}{\sin x}(1+\cos x)$$

$$=\lim_{x\to 0}\frac{x}{\sin x}\frac{3\frac{\sin 3x}{3x}}{\frac{\sin x}{x}}(1+\cos x)=(1)\frac{3(1)}{1}(2)=6$$

16) $\lim_{x\to 0}\frac{1-\cos x+\sin x}{x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}+\sin x}{x}$

$$= \lim_{x\to 0^-} \frac{2x-\sin x}{-\sin x}\frac{\sqrt{1+\cos x}}{1}$$

$$\lim_{x\to 0^-}\left(-2\frac{x}{\sin x}+1\right)\frac{\sqrt{1+\cos x}}{1} = (-2(1)+1)(\sqrt{2})$$
$$= -\sqrt{2}$$

22) $$\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{\tan^2 x} = \lim_{x\to 0}\frac{1-\sqrt{\cos x}}{\tan^2 x}\frac{1+\sqrt{\cos x}}{1+\sqrt{\cos x}}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{\tan^2 x}\frac{1+\cos x}{1+\cos x}\frac{1}{1+\sqrt{\cos x}}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{\tan^2 x}\frac{1}{(1+\sqrt{\cos x})(1+\cos x)}$$

$$= \lim_{x\to 0}\frac{\sin^2 x}{\tan^2 x}\frac{1}{(1+\sqrt{\cos x})(1+\cos x)}$$

$$= \lim_{x\to 0}\cos^2 x\ \frac{1}{(1+\sqrt{\cos x})(1+\cos x)} = 1^2\left(\frac{1}{4}\right) = \frac{1}{4}$$

23) $$\lim_{x\to 0}\frac{1+x-\cos 2x}{1-\sqrt{1+\sin x}}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x+x}{1-\sqrt{1+\sin x}}\left(\frac{1+\sqrt{1+\sin x}}{1+\sqrt{1+\sin x}}\right)$$

$$= \lim_{x\to 0}\frac{2\sin^2 x+x}{1-1-\sin x}\left(\frac{1+\sqrt{1+\sin x}}{1}\right)$$

$$= \lim_{x\to 0}\left(\frac{2\sin^2 x+x}{-\sin x}\right)\left(\frac{1+\sqrt{1+\sin x}}{1}\right)$$

$$= \lim_{x\to 0}\left(-2\sin x-\frac{x}{\sin x}\right)\left(\frac{1+\sqrt{1+\sin x}}{1}\right)$$
$$= (0-1)(2) = -2$$

24) $$\lim_{x\to 0}\frac{(a-b)x}{\sin ax-\sin bx}$$

نقسم البسط و المقام على $x$ :

$$\lim_{x\to 0}\frac{(a-b)x}{\sin ax-\sin bx} = \lim_{x\to 0}\frac{a-b}{\frac{\sin ax}{x}-\frac{\sin bx}{x}}$$

$$= \lim_{x\to 0}\frac{a-b}{a\frac{\sin ax}{ax}-b\frac{\sin bx}{bx}} = \frac{a-b}{a(1)-b(1)} = 1$$

$$= \lim_{x\to 0}\frac{1-\cos x}{\tan^2 x}\frac{1+\cos x}{1+\cos x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{\tan^2 x}\frac{1}{1+\cos x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{\sin^2 x}{\tan^2 x}\frac{1}{1+\cos x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\cos^2 x\ \frac{1}{1+\cos x}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= 1^2\ \frac{1}{1+1}\frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

20) $$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{\sin 2x}$$

$$= \lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{\sin 2x}\frac{\sqrt{1+\tan x}+\sqrt{1-\tan x}}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}$$

$$= \lim_{x\to 0}\frac{1+\tan x-1+\tan x}{\sin 2x}\frac{1}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}$$

$$= \lim_{x\to 0}\frac{2\tan x}{2\sin x\cos x}\frac{1}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}$$

$$= \lim_{x\to 0}\frac{1}{\cos^2 x}\frac{1}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}$$

$$= (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

21) $$\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos^2 x}}\frac{\sqrt{1+\cos x}}{1}$$

$$= \lim_{x\to 0}\frac{2x-\sin x}{\sqrt{\sin^2 x}}\frac{\sqrt{1+\cos x}}{1} = \lim_{x\to 0}\frac{2x-\sin x}{|\sin x|}\frac{\sqrt{1+\cos x}}{1}$$

من أجل $x\to 0^+$ نجد أن $|\sin x| = \sin x$

$$\lim_{x\to 0^+}\frac{2x-\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0^+}\frac{2x-\sin x}{\sin x}\frac{\sqrt{1+\cos x}}{1}$$

$$= \lim_{x\to 0^+}\left(2\frac{x}{\sin x}-1\right)\frac{\sqrt{1+\cos x}}{1} = (2(1)-1)(\sqrt{2}) = \sqrt{2}$$

أما من أجل $x\to 0^-$ نجد أن $|\sin x| = -\sin x$

$$\lim_{x\to 0^-}\frac{2x-\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0^-}\frac{2x-\sin x}{-\sin x}\frac{\sqrt{1+\cos x}}{1}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x\,(2\cos^2 x-1)}{x^2} \cdot \frac{1}{1+\cos x\sqrt{\cos 2x}}$$

$$= \lim_{x\to 0} \frac{1-2\cos^4 x+\cos^2 x}{x^2} \cdot \frac{1}{1+\cos x\sqrt{\cos 2x}}$$

ليكن $cos\,x = t$

$$1+t^2-2t^4=0$$

$$t_1 = 1\ , \qquad t_2 = -1$$

بإجراء القسمة الإقليدية نجد:

$$1+t^2-2t^4 = (1-t)(1+t)(2t^2+1)$$

$$\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)(2\cos^2 x+1)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}(1+\cos x)(2\cos^2 x+1)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$= \lim_{x\to 0} 2\left(\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\right)^2 \frac{(1+\cos x)(2\cos^2 x+1)}{\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$= (2)(\frac{1}{2}(1))^2\frac{6}{2} = \frac{3}{2}$$

28) $$\lim_{x\to 0} \frac{x\sin 2x}{1-\cos x} = \lim_{x\to 0} \frac{2x\sin x\cos x}{1-\cos x}\,\frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to 0} \frac{2x\sin x\cos x}{1-\cos^2 x}\,\frac{1+\cos x}{1}$$

$$= \lim_{x\to 0} \frac{2x\sin x\cos x}{\sin^2 x}\,\frac{1+\cos x}{1}$$

$$= \lim_{x\to 0} \frac{2x\cos x}{\sin x}\,\frac{1+\cos x}{1} = (2)(1)(1)(2) = 4$$

29) $$\lim_{x\to 0} \frac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{\cos^2 x-\cos 2x}{\sin^2 x\,(\cos x+\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0} \frac{\cos^2 x-(\cos^2 x-\sin^2 x)}{\sin^2 x\,(\cos x+\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{\sin^2 x\,(\cos x+\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0} \frac{1}{(\cos x+\sqrt{\cos 2x})} = \frac{1}{2}$$

25) $$\lim_{x\to 0} \frac{1-\cos x\cos 2x}{x^2}$$

**$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$**

$$\lim_{x\to 0} \frac{1-\cos x\cos 2x}{x^2} = \lim_{x\to 0} \frac{1-\cos x\,(2\cos^2 x-1)}{x^2}$$

$$= \lim_{x\to 0} \frac{1+\cos x-2\cos^3 x}{x^2}$$

ليكن $cos\,x = t$

$$1+t-2t^3=0$$

$$t_0 = 1$$

بإجراء القسمة الإقليدية نجد:

$$1+t-2t^3 = (1-t)(2t^2+2t+1)$$

$$\lim_{x\to 0} \frac{1-\cos x\cos 2x}{x^2}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(2\cos^2 x+2\cos x+1)}{x^2}$$

$$= \lim_{x\to 0} \frac{(2\sin^2\frac{x}{2})(2\cos^2 x+2\cos x+1)}{x^2}$$

$$= \lim_{x\to 0} \frac{(2\sin^2\frac{x}{2})(2\cos^2 x+2\cos x+1)}{x^2}$$

$$= \lim_{x\to 0} 2\left(\frac{\sin\frac{x}{2}}{x}\right)^2 (2\cos^2 x+2\cos x+1)$$

$$= \lim_{x\to 0} 2\left(\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\right)^2 (2\cos^2 x+2\cos x+1)$$

$$= (2)(\frac{1}{2}(1))^2(5) = \frac{5}{2}$$

26) $$\lim_{x\to 0} \frac{\cos^2 x-\cos x}{x^2} = \lim_{x\to 0} \frac{1-\cos x-\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}-\sin^2 x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\right)^2 - \left(\frac{\sin x}{x}\right)^2$$

$$= \frac{1}{2}-1 = \frac{-1}{2}$$

27) $$\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2} \cdot \frac{1+\cos x\sqrt{\cos 2x}}{1+\cos x\sqrt{\cos 2x}}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{x^2} \cdot \frac{1}{1+\cos x\sqrt{\cos 2x}}$$

$$= \lim_{x\to\frac{\pi}{2}} \sqrt[3]{(1+\sin x)\cos x} = \sqrt[3]{2\times 0} = 0$$

36) $$\lim_{x\to 0}\frac{x^2-1+\cos^2 x}{x\sin x} = \lim_{x\to 0}\frac{x^2-\sin^2 x}{x\sin x}$$

$$= \lim_{x\to 0}\frac{1-\left(\frac{\sin x}{x}\right)^2}{\frac{\sin x}{x}} = \frac{1-1}{1} = 0$$

37) $$\lim_{x\to\frac{\pi}{2}}\frac{\sin x\ -1}{x-\frac{\pi}{2}}$$

ليكن التابع $f$ المعرف على $\mathbb{R}$ وفق :

$$f(x) = \sin x$$

$$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$$

$$f'(x) = \cos x$$

$$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin x\ -1}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}}\frac{f(x)-f\left(\frac{\pi}{2}\right)}{x-\frac{\pi}{2}} = f'\left(\frac{\pi}{2}\right) = 0$$

حسب تعريف العدد المشتق .

**-انتهى الحل-**

**لطرح أسئلتكم و استفساراتكم التواصل عبر المعرف :**

**https://telegram.me/ABDULMALEK111**

**للمزيد من النماذج انضموا إلى القناة**

**https://telegram.me/BAC_MATHS_1**

30) $$\lim_{x\to 0}\frac{x}{x+\sin x} = \lim_{x\to 0}\frac{1}{1+\frac{\sin x}{x}} = \frac{1}{2}$$

31) $$\lim_{x\to 0}\frac{\sin x}{\sqrt{x}} = \lim_{x\to 0}\frac{\sin x}{x}\sqrt{x} = (1)(0) = 0$$

32) $$\lim_{x\to 0}\frac{\sin^2 9x-x^2}{1-\cos 2x} = \lim_{x\to 0}\frac{\sin^2 9x-x^2}{2\sin^2 x}$$

نقسم البسط و المقام على $x^2$ :

$$\lim_{x\to 0}\frac{\sin^2 9x-x^2}{1-\cos 2x} = \lim_{x\to 0}\frac{\frac{\sin^2 9x}{x^2}-1}{2\frac{\sin^2 x}{x^2}}$$

$$= \lim_{x\to 0}\frac{\left(9\frac{\sin 9x}{9x}\right)^2-1}{2\left(\frac{\sin x}{x}\right)^2} = \frac{81(1)-1}{2(1)} = 40$$

33) $$\lim_{x\to 0}\frac{1-\cos x}{\sin x} = \lim_{x\to 0}\frac{\sin^2 x}{\sin x\,(1+\cos x)}$$

$$\lim_{x\to 0}\frac{\sin x}{(1+\cos x)} = \frac{0}{2} = 0$$

34) $$\lim_{x\to-\frac{\pi}{4}}\frac{\sin x+\cos x}{x+\frac{\pi}{4}}$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

$$\lim_{x\to-\frac{\pi}{4}}\frac{\sin x+\cos x}{x+\frac{\pi}{4}} = \lim_{x\to-\frac{\pi}{4}}\sqrt{2}\frac{\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x}{x+\frac{\pi}{4}}$$

$$= \lim_{x\to-\frac{\pi}{4}}\sqrt{2}\frac{\cos\frac{\pi}{4}\sin x+\sin\frac{\pi}{4}\cos x}{x+\frac{\pi}{4}}$$

$$= \lim_{x\to-\frac{\pi}{4}}\sqrt{2}\frac{\sin(x+\frac{\pi}{4})}{x+\frac{\pi}{4}} = \sqrt{2}(1) = \sqrt{2}$$

35) $$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\sqrt[3]{(1-\sin x)\frac{1+\sin x}{1+\sin x}}}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\sqrt[3]{(1-\sin^2 x)\frac{1}{1+\sin x}}} = \lim_{x\to\frac{\pi}{2}}\frac{\sqrt[3]{1+\sin x}\cos x}{\sqrt[3]{\cos^2 x}}$$