

اسم الطالب: \_\_\_\_\_

الشعبة: \_\_\_\_\_

1- إذا كان  $\beta$  عدداً حقيقياً وكان العدد العقدي  $w = \frac{\beta+i\sqrt{3}}{\sqrt{3}-i\beta}$  فإن  $|w|$  تساوي:

$\beta^2 + 3$	D	1	C	$\sqrt{3}$	B	$\frac{1}{\sqrt{3}}$	A
---------------	---	---	---	------------	---	----------------------	---

2- مجموعة النقط المستوي  $M(Z)$  التي تحقق أن  $|Z - 2 + i| = 5$ :

دائرة مركزها $\Omega(2, -1)$ $R = \sqrt{5}$	D	دائرة مركزها $\Omega(2, -1)$ $R = 5$	C	دائرة مركزها $\Omega(-2, 1)$ $R = 5$	B	محور قطعة مستقيمة	A
--	---	---	---	---	---	-------------------	---

3- أحد حلول المعادلة  $j^3 = 1$  هو:

$e^{\pi i}$	D	$1 + i$	C	$e^{2\frac{\pi}{3}i}$	B	-1	A
-------------	---	---------	---	-----------------------	---	----	---

4-  $P(Z) = Z^3 - 2(a + i\sqrt{3})Z^2 - 4(a - i\sqrt{3})Z + 8 = 0$  قيمة  $a$  لكي يكون  $Z = 2$  حلاً للمعادلة  $P(Z) = 0$

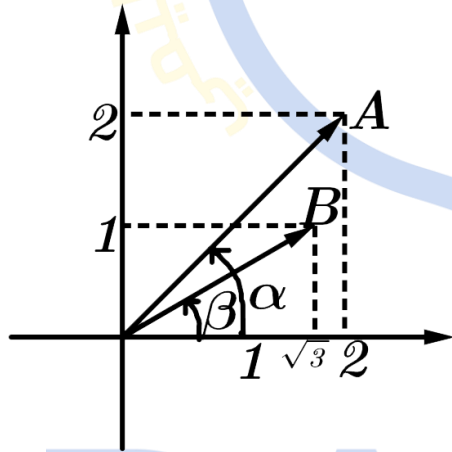
1	D	-16	C	8	B	-8	A
---	---	-----	---	---	---	----	---

5- لتكن  $A, B, C$  نقاط المستوي التي تمثل الأعداد العقدية بالترتيب  $a = 2, b = 1 + i\sqrt{3}, c = -1 + i\sqrt{3}$  فإن قيمة الكسر  $\frac{a-b}{c-b}$  تساوي:

1	D	$e^{-\frac{\pi}{3}i}$	C	$e^{\frac{\pi}{3}i}$	B	$e^{2\frac{\pi}{3}i}$	A
---	---	-----------------------	---	----------------------	---	-----------------------	---

6- نتأمل المعلم العقدي المتجانس  $(o, \vec{u}, \vec{v})$  بفرض  $\alpha$  القياس الأساسي للزاوية  $(\vec{u}, \vec{oA})$

و  $\beta$  القياس الأساسي للزاوية  $(\vec{u}, \vec{oB})$  فإن قيمة  $(\alpha - \beta)$  تساوي:



$\frac{\pi}{12}$	D	$\frac{\pi}{3}$	C	$\frac{\pi}{6}$	B	$\frac{\pi}{4}$	A
------------------	---	-----------------	---	-----------------	---	-----------------	---

7- ليكن العدد العقدي  $w = \frac{-\sqrt{2}}{1+i} e^{i\frac{\pi}{3}}$  فإن  $w$  بالشكل الأسّي:

$e^{i\frac{11\pi}{12}}$	D	$-e^{i\frac{\pi}{12}}$	C	$e^{i\frac{\pi}{12}}$	B	$e^{i\frac{13\pi}{12}}$	A
-------------------------	---	------------------------	---	-----------------------	---	-------------------------	---

8- ليكن العدد العقدي  $w = \frac{-1}{1+i}$  فإن  $arg(Z)$ :

$\frac{\pi}{12}$	D	$\frac{5\pi}{4}$	C	$\frac{3\pi}{4}$	B	$\frac{\pi}{4}$	A
------------------	---	------------------	---	------------------	---	-----------------	---



9- ليكن  $w, z$  عدنان عقديان بحيث  $|w| = 1$  وليكن  $Z = \frac{z - \bar{z}w}{1 - w}$  فإن:

$ Z  = 1$	<b>A</b>	$arg(Z) = \frac{\pi}{2}$	<b>B</b>	$Z$ حقيقي	<b>C</b>	$Z$ تخيلي بحت	<b>D</b>
-----------	----------	--------------------------	----------	-----------	----------	---------------	----------

10- إذا كان  $Z_1 = -1 + i$  هو حلاً للمعادلة  $P(Z) = Z^2 + (1 + 2i)Z + 3 + 3i$  فإن الحل الآخر للمعادلة هو:

$Z_2 = -1 - i$	<b>A</b>	$Z_2 = 1 - i$	<b>B</b>	$Z_2 = -3i$	<b>C</b>	$Z_2 = 3i$	<b>D</b>
----------------	----------	---------------	----------	-------------	----------	------------	----------

11- في المستوي العقدي المنسوب إلى معلم متجانس  $(o, \vec{u}, \vec{v})$  تتأمل النقاط  $M, D, C$   $M = -1 + i, C = 2i, D = -2$  فإن  $arg \frac{c-d}{m}$  تساوي:

$\frac{\pi}{2}$	<b>A</b>	$-\frac{\pi}{2}$	<b>B</b>	0	<b>C</b>	$\pi$	<b>D</b>
-----------------	----------	------------------	----------	---	----------	-------	----------

12- في المستوي العقدي المنسوب إلى معلم متجانس  $(o, \vec{u}, \vec{v})$  تتأمل النقطتين  $A, B$  اللتان يمثلهما على الترتيب العدنان العقديان  $Z_A = 4, Z_B = 2\sqrt{2} + 2\sqrt{2}i$  ولتكن  $I$  منتصف  $[AB]$  فإن قياس الزاوية  $(\vec{u}, \vec{oI})$  تساوي:

$\frac{\pi}{4}$	<b>A</b>	$\pi$	<b>B</b>	$\frac{\pi}{8}$	<b>C</b>	$\frac{3\pi}{4}$	<b>D</b>
-----------------	----------	-------	----------	-----------------	----------	------------------	----------

13- ليكن العدد العقدي  $Z = 2e^{\frac{3\pi}{4}i}$  يكتب بالشكل الجبري:

$Z = -1 + i$	<b>A</b>	$Z = -\sqrt{2} - i\sqrt{2}$	<b>B</b>	$Z = \sqrt{2} - i\sqrt{2}$	<b>C</b>	$Z = -\sqrt{2} + i\sqrt{2}$	<b>D</b>
--------------	----------	-----------------------------	----------	----------------------------	----------	-----------------------------	----------

14- ليكن العدد العقدي  $Z = (1 + i)^9$  فإنه يكتب بالشكل:

$Z = 16(1 + i)$	<b>A</b>	$Z = 1 + i$	<b>B</b>	$Z = -1 + i$	<b>C</b>	$Z = 8(1 + i)$	<b>D</b>
-----------------	----------	-------------	----------	--------------	----------	----------------	----------

15- ليكن العدنان العقديان  $Z_1 = 1 + \sqrt{3}i$  و  $Z_2 = 1 + i$  فإن الشكل المثلثي ل  $\frac{Z_1}{Z_2}$  يساوي:

$\sqrt{2} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$	<b>A</b>	$\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}$	<b>B</b>
$\sqrt{2} \left( \cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$	<b>C</b>	$\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}$	<b>D</b>

16- لتكن النقطة  $A$  الممثلة بالعدد العقدي  $Z_A = -1 + i$  فإن  $Z_A$  بالشكل الأسّي:

$2e^{\frac{3\pi}{4}i}$	<b>A</b>	$\sqrt{2}e^{\frac{3\pi}{4}i}$	<b>B</b>	$2e^{\frac{5\pi}{4}i}$	<b>C</b>	$\sqrt{2}e^{\frac{5\pi}{4}i}$	<b>D</b>
------------------------	----------	-------------------------------	----------	------------------------	----------	-------------------------------	----------

17- إذا كان  $2 + i$  هو أحد جذري المعادلة  $Z^2 - bZ + 5 = 0$  فإن قيمة  $b$  تساوي:

$b = 2$	<b>A</b>	$b = 4$	<b>B</b>	$b = -2$	<b>C</b>	$b = -4$	<b>D</b>
---------	----------	---------	----------	----------	----------	----------	----------

18- مجموعة النقط  $M(x, y)$  التي تحقق  $Z \cdot \bar{Z} = 4$ :

قرص دائري مركزه $(o)$ و $R = 2$	<b>A</b>	دائرة مركزها $(o)$ و $R = 2$	<b>B</b>
قطع زائد متساوي الساقين	<b>C</b>	محور قطعة مستقيمة	<b>D</b>

19-  $e^{i\theta}$  يساوي:

$x + iy$	<b>A</b>	$x - iy$	<b>B</b>	$\sqrt{x^2 + y^2}(x + iy)$	<b>C</b>	$\frac{x}{\sqrt{x^2 + y^2}} + i \frac{y}{\sqrt{x^2 + y^2}}$	<b>D</b>
----------	----------	----------	----------	----------------------------	----------	---	----------



20- ليكن العدد العقدي  $Z = 1 + e^{2i\theta}$  حيث  $\theta \in ]\frac{\pi}{2}, \pi[$ :

$-2\cos\theta e^{i\theta}$	D	$2\cos\theta e^{i\theta}$	C	$2\sin\theta e^{i\theta}$	B	$2\tan\theta e^{i\theta}$	A
----------------------------	---	---------------------------	---	---------------------------	---	---------------------------	---

21- للعددين  $Z$  و  $\bar{Z}$  صورتان في المستوي العقدي متناظرتان بالنسبة لـ:

منصف الربعين 3 و 1	D	المبدأ	C	محور $yy'$	B	محور $xx'$	A
--------------------	---	--------	---	------------	---	------------	---

22-  $Z = \frac{1+i\tan\theta}{1-i\tan\theta}$  بحيث  $\theta \in ]-\frac{\pi}{2}, \frac{\pi}{2}[$  يكتب بالشكل الأسّي:

$Z = e^{i\theta}$	D	$Z = e^{2i\theta}$	C	$Z = 2e^{i\theta}$	B	$Z = 2e^{2i\theta}$	A
-------------------	---	--------------------	---	--------------------	---	---------------------	---

23-  $(\frac{\sqrt{3}}{2} - i\frac{1}{2})^{24}$  تساوي:

$-i$	D	$i$	C	1	B	$-1$	A
------	---	-----	---	---	---	------	---

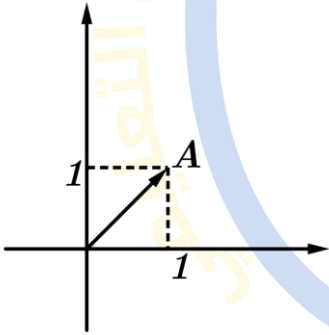
24- لتكن  $Z = 1 - i$  عندئذ قيمة  $Im(\frac{1}{Z})$  هي:

$-\frac{1}{2}$	D	$\frac{1}{2}$	C	+1	B	$\frac{1}{2}i$	A
----------------	---	---------------	---	----	---	----------------	---

25- العدد العقدي  $Z = i + \frac{1-i}{1+i}$  يساوي:

$2i$	D	0	C	1	B	$-1$	A
------	---	---	---	---	---	------	---

26- يكتب العدد  $Z_A$  بالشكل الأسّي:



$\sqrt{2}e^{-\frac{\pi}{4}i}$	D	$\sqrt{2}e^{\frac{3\pi}{4}i}$	C	$2e^{\frac{\pi}{4}i}$	B	$\sqrt{2}e^{\frac{\pi}{4}i}$	A
-------------------------------	---	-------------------------------	---	-----------------------	---	------------------------------	---

27- إذا كان  $\sqrt{2}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}) = \frac{\sqrt{3}+1}{2} + i\frac{\sqrt{3}-1}{2}$  فإن:

$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{2}$	D	$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$	C	$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2}$	B	$\sin\frac{\pi}{12} = \frac{\sqrt{3}+1}{2}$	A
--	---	--	---	---	---	---	---

28- ليكن  $Z = (1 - \sqrt{2})(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$  فإنه يكتب بالشكل الأسّي:

$(\sqrt{2}-1)e^{\frac{4\pi}{3}i}$	D	$(\sqrt{2}-1)e^{\frac{\pi}{3}i}$	C	$(\sqrt{2}-1)e^{\frac{2\pi}{3}i}$	B	$(1-\sqrt{2})e^{\frac{\pi}{3}i}$	A
-----------------------------------	---	----------------------------------	---	-----------------------------------	---	----------------------------------	---

29- ليكن  $Z = 2i(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$  فإنه يكتب بالشكل الأسّي:

$2e^{\frac{\pi}{6}i}$	D	$2e^{\frac{2\pi}{3}i}$	C	$2e^{\frac{\pi}{3}i}$	B	$2e^{\frac{5\pi}{6}i}$	A
-----------------------	---	------------------------	---	-----------------------	---	------------------------	---

30- ليكن  $Z = -2(\sin\frac{\pi}{3} + i\cos\frac{\pi}{3})$  فإنه يكتب بالشكل الأسّي:

$2e^{\frac{\pi}{3}i}$	D	$2e^{\frac{5\pi}{6}i}$	C	$2e^{\frac{7\pi}{6}i}$	B	$2e^{\frac{\pi}{6}i}$	A
-----------------------	---	------------------------	---	------------------------	---	-----------------------	---



31- ليكن  $Z = 2(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$  يكتب بالشكل الجبري:

$\sqrt{3} - i$	D	$-\sqrt{3} + i$	C	$-\sqrt{3} - i$	B	$-1 + i\sqrt{3}$	A
----------------	---	-----------------	---	-----------------	---	------------------	---

32- الشكل الجبري للعدد العقدي  $Z = (1 + i)^{2025}$ :

$2^{1012}(1 + i)$	D	$2^{1012}$	C	$2^{1012}i$	B	$2(1 + i)^{1012}$	A
-------------------	---	------------	---	-------------	---	-------------------	---

33- يكون  $w = (z + 1)(\bar{z} - 2)$  حقيقياً إذا كان:

$z = 1 + i$	D	$ z  = 1$	C	$z$ تخيلي بحت	B	$z$ حقيقي	A
-------------	---	-----------	---	---------------	---	-----------	---

34- إذا كان جذر للمعادلة  $Z^4 - 6Z^3 + 24Z^2 - 18Z + 63 = 0$  فإن الجذر الآخر هو:

$1 + i\sqrt{3}$	D	$-\sqrt{3}i$	C	$1 - i\sqrt{3}$	B	1	A
-----------------	---	--------------	---	-----------------	---	---	---

35- أحد حلول المعادلة  $Z^2 = i$  هو:

$\sqrt{2} + i\sqrt{2}$	D	$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	C	$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$	B	1 + i	A
------------------------	---	--	---	--	---	-------	---

36- إذا كان  $Z_1 = 1 + i$  حلاً للمعادلة  $Z^2 + pZ + q = 0$  بحيث  $p, q \in \mathbb{R}$  فإن:

$p = -2, q = -2$	D	$p = 2, q = -2$	C	$p = -2, q = 2$	B	$p = 2, q = 2$	A
------------------	---	-----------------	---	-----------------	---	----------------	---

37- ليكن  $\theta$  عدداً من المجال  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  فإن الشكل الأسّي للعدد العقدي  $Z = 1 + e^{2i\theta}$ :

$\cos 2\theta \cdot e^{2i\theta}$	D	$2\cos 2\theta \cdot e^{i\theta}$	C	$2\cos \theta \cdot e^{i\theta}$	B	$2\cos \theta \cdot e^{2i\theta}$	A
-----------------------------------	---	-----------------------------------	---	----------------------------------	---	-----------------------------------	---

38- مجموعة النقاط  $M_Z$  التي تمثلها العلاقة  $(z + \bar{z})^2 - (z - \bar{z})^2 = 4$ :

دائرة مركزها $(0)$ و $R = 2$	D	دائرة مركزها $(0)$ و $R = 1$	C	مستقيم أفقي	B	مستقيم شاقولي	A
------------------------------	---	------------------------------	---	-------------	---	---------------	---

39- أيّاً كان العدد الحقيقي  $\theta$  فإن  $e^{i\frac{\theta}{2}} - e^{-i\frac{\theta}{2}}$  تساوي:

$2 \sin \frac{\theta}{2}$	D	$2i \sin \frac{\theta}{2}$	C	$2 \cos \frac{\theta}{2}$	B	$2i \sin \theta$	A
---------------------------	---	----------------------------	---	---------------------------	---	------------------	---

40- إن مقلوب العدد العقدي  $Z = a + ib$  حيث  $Z \neq 0$  هو العدد:

$\frac{1}{a} + i\frac{1}{b}$	B	$\frac{a}{a^2 + b^2} + i\frac{-b}{a^2 + b^2}$	A
$\frac{a}{a^2 - b^2} + i\frac{b}{a^2 - b^2}$	D	$\frac{a}{a^2 - b^2} + i\frac{-b}{a^2 - b^2}$	C

انتهت الأسئلة



حلول أسئلة المذاكرة

(5)

$$\frac{a-b}{c-b} = \frac{2-1-i\sqrt{3}}{-1+i\sqrt{3}-1-i\sqrt{3}}$$

$$= \frac{1-i\sqrt{3}}{-2} = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{\frac{2\pi}{3}i}$$

A

(6)

$$Z_A = 2 + 2i \quad \arg(Z_A) = \frac{\pi}{4}$$

$$Z_B = \sqrt{3} + i \quad \arg(Z_B) = \frac{\pi}{6}$$

$$\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$$

D

(7)

$$w = \frac{-\sqrt{2}}{1+i} e^{\frac{\pi}{3}i} = \frac{-\sqrt{2}}{\sqrt{2}e^{\frac{\pi}{4}i}} e^{\frac{\pi}{3}i}$$

$$w = -e^{-\frac{\pi}{4}} \cdot e^{\frac{\pi}{3}i} = -e^{\frac{\pi}{12}i}$$

$$w = e^{\frac{13\pi}{12}i}$$

A

(8)

$$w = \frac{-1}{\sqrt{2}e^{\frac{\pi}{4}i}} = \frac{-1}{\sqrt{2}} e^{-\frac{\pi}{4}i}$$

$$w = \frac{1}{\sqrt{2}} e^{\pi i} \cdot e^{-\frac{\pi}{4}i}$$

$$w = \frac{1}{\sqrt{2}} e^{\frac{3\pi}{4}i}$$

B

(1)

$$|w| = \left| \frac{\beta + i\sqrt{3}}{\sqrt{3} - i\beta} \right| = \left| \frac{\beta + i\sqrt{3}}{\sqrt{3} - i\beta} \right|$$

$$|w| = \frac{\sqrt{\beta^2 + 3}}{\sqrt{3 + \beta^2}} = 1$$

C

(2)

$$Z = x + iy \quad \text{بفرض}$$

$$|x + iy - 2 + i| = 5$$

$$|(x-2) + i(y+1)| = 5$$

$$\sqrt{(x-2)^2 + (y+1)^2} = 5$$

$$(x-2)^2 + (y+1)^2 = 25 \quad \text{بالتربيع}$$

وهي معادلة دائرة مركزها  $\Omega(2, -1)$  و  $R = 5$

C

(3)

$$\text{بفرض } j = e^{i\theta} \Rightarrow e^{3i\theta} = e^{i\theta}$$

$$3\theta = 0 + 2\pi k \Rightarrow \theta = \frac{2}{3}\pi k$$

$$k = 0 \Rightarrow j_1 = e^{0i} \quad \text{الحل الأول}$$

$$k = 1 \Rightarrow j_2 = e^{\frac{2}{3}\pi i} \quad \text{الحل الثاني}$$

$$k = 2 \Rightarrow j_3 = e^{2\pi i} \quad \text{الحل الثالث}$$

أو ممكن نستخدم التجريب

B

(4)

نعوض  $Z = 2$  في المعادلة:

$$8 - 2(a + i\sqrt{3})(4) - 4(a - i\sqrt{3})(2) + 8 = 0$$

$$8 - 8a - 8\sqrt{3}i - 8a + 8\sqrt{3}i + 8 = 0$$

$$-16a = -16 \Rightarrow a = 1$$

D



$$\widehat{BOA} = (\vec{u}, \vec{OB}) = \frac{\pi}{4}$$

$$(\vec{u}, \vec{OI}) = \frac{\pi}{8}$$

نصف  $(\vec{u}, \vec{OB})$

(13)

$$Z = 2e^{\frac{3\pi}{4}i} = 2 \left[ \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right]$$

$$= 2 \left[ \cos \left( \pi - \frac{\pi}{4} \right) + i \sin \left( \pi - \frac{\pi}{4} \right) \right]$$

$$= \left[ -\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$$

$$= -\sqrt{2} + i\sqrt{2}$$

(14)

$$Z = (1+i)^9 = (1+i)(1+i)^8$$

$$Z = (1+i)((1+i)^2)^4$$

$$Z = (1+i)(2i)^4 = 16(1+i)$$

A

(15)

$$\frac{Z_1}{Z_2} = \frac{1+i\sqrt{3}}{1+i}$$

$$\frac{Z_1}{Z_2} = \frac{2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{\sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)}$$

$$\frac{Z_1}{Z_2} = \sqrt{2} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

A

(9)

$$\bar{Z} = \frac{\bar{z} - z\bar{w}}{1 - \bar{w}}$$

$$|w| = 1 \Rightarrow \bar{w} = \frac{1}{w}$$

$$\bar{z} - \frac{z}{w} = \frac{w\bar{z} - z}{w}$$

$$\bar{Z} = \frac{w\bar{z} - z}{w - 1} = \frac{z - w\bar{z}}{1 - w} = Z$$

ومنه Z حقيقي

C

(10)

$$Z_1 + Z_2 = \frac{-b}{a}$$

$$-1 + i + Z_2 = -1 - 2i$$

$$Z_2 = -3i$$

C

(11)

$$\frac{c-d}{m} = \frac{2i+2}{-1+i} = 2 \left( \frac{1+i}{-1+i} \right)$$

$$= 2 \frac{(1+i)(-1-i)}{1+1} = -2(1+i)^2$$

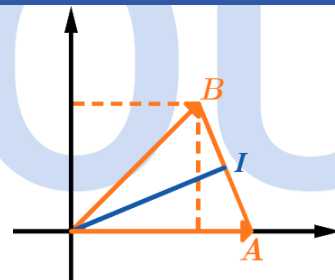
$$= -2 \cdot 2i = -4i$$

$$= e^{\pi i} \cdot 4 \cdot e^{\frac{\pi}{2}i} = 4e^{\frac{3\pi}{2}i}$$

$$\arg \frac{c-d}{c} = \frac{3\pi}{2} = -\frac{\pi}{2}$$

B

(12)



$$|B| = 4$$

$$|A| = 4$$

فالمثلث OAB متساوي الساقين فيه OI متوسط فهو

منصف



(21)

$$\left. \begin{aligned} Z &= x + iy \\ Z &= x - iy \end{aligned} \right\} \text{متناظران بالنسبة لـ } Ox$$

A

(22)

$$\frac{1 + i \tan \theta}{1 - i \tan \theta} = \frac{1 + i \frac{\sin \theta}{\cos \theta}}{1 - i \frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\cos \theta + i \sin \theta}{\cos \theta - i \sin \theta} = \frac{e^{i\theta}}{e^{-i\theta}} = e^{2i\theta}$$

C

(23)

$$\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{24} = \left(e^{-\frac{\pi}{6}i}\right)^{24} = e^{-24\frac{\pi}{6}i}$$

$$= e^{-4\pi i} = e^{0i} = 1$$

B

(24)

$$Z = 1 - i \Rightarrow \frac{1}{Z} = \frac{1}{1 - i} = \frac{1 + i}{1 + 1}$$

$$\frac{1}{Z} = \frac{1}{2} + \frac{1}{2}i$$

$$\text{Im}\left(\frac{1}{Z}\right) = \frac{1}{2}$$

C

(25)

$$Z = i + \frac{(1 + i)^2}{1 + 1} = i + \frac{-2i}{2}$$

$$Z = i - i = 0$$

C

(16)

$$Z_A = -1 + i = \sqrt{2}e^{\frac{3\pi}{4}i}$$

B

(17)

$$(2 + i)^2 - b(2 + i) + 5 = 0$$

$$4 + 4i - 1 - 2b - bi + 5 = 0$$

$$8 - 2b + i(4 - b) = 0 + 0i$$

بالمطابقة:

$$8 - 2b = 0 \Rightarrow b = 4$$

$$4 - b = 0 \Rightarrow b = 4$$

B

(18)

$$Z \cdot \bar{Z} = 4 \Leftrightarrow x^2 + y^2 = 4$$

وهي معادلة دائرة مركزها  $\Omega(0, 0)$  و  $R = 2$

B

(19)

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{i\theta} = \frac{x}{r} + i \frac{y}{r}$$

$$e^{i\theta} = \frac{x}{\sqrt{x^2 + y^2}} + i \frac{y}{\sqrt{x^2 + y^2}}$$

D

(20)

$$Z = 1 + e^{2i\theta} = e^{i\theta} \cdot e^{-i\theta} + e^{i\theta} \cdot e^{i\theta}$$

$$= (e^{i\theta} + e^{-i\theta})e^{i\theta} = 2\cos \theta \cdot e^{i\theta}$$

$$= -2\cos \theta \cdot e^{i\theta}$$

$$\theta \in \left] \frac{\pi}{2}, \pi \right[ \Rightarrow \cos \theta < 0$$

D



(31)

$$\begin{aligned} Z &= 2 \left[ \cos \left( \pi - \frac{\pi}{6} \right) + i \sin \left( \pi - \frac{\pi}{6} \right) \right] \\ &= 2 \left[ -\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right] \\ &= 2 \left[ -\frac{\sqrt{3}}{2} + i \frac{1}{2} \right] = -\sqrt{3} + i \end{aligned}$$

C

(32)

$$\begin{aligned} (1 + i)^{2025} &= (1 + i)(1 + i)^{2024} \\ &= (1 + i)((1 + i)^2)^{1012} \\ &= (1 + i)(2i)^{1012} \\ &= (1 + i) \cdot 2^{1012} \cdot i^{2012} \\ &= (1 + i) \cdot 2^{1012} \cdot (i^2)^{506} \\ &= 2^{1012}(1 + i) \end{aligned}$$

D

(33)

$$w \text{ حقيقي} \Leftrightarrow \bar{w} = w$$

$$\begin{aligned} (\bar{z} + 1)(z - 2) &= (z + 1)(\bar{z} - 2) \\ 3\bar{z} - 2\bar{z} + z - 2 &= 3\bar{z} - 2z + \bar{z} - 2 \\ -2\bar{z} + z &= -2z + \bar{z} \\ -3\bar{z} &= -3z \Leftrightarrow \bar{z} = z \end{aligned}$$

A

z حقيقي

(34)

بما أن المعاملات حقيقية فالجذر الثاني هو مرافق الجذر الأول

$$Z_2 = -i\sqrt{3}$$

C

(35)

$$\left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right)^2 = i \text{ لأن } \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

C

ويمكن استخدام المعادلات الأساسية

(26)

$$Z_A = 1 + i = \sqrt{2}e^{\frac{\pi}{4}i}$$

A

(27)

$$\begin{aligned} \sqrt{2} \cos \frac{\pi}{12} + i\sqrt{2} \sin \frac{\pi}{12} \\ = \frac{\sqrt{3} + 1}{2} + i \frac{\sqrt{3} - 1}{2} \end{aligned}$$

$$\sqrt{2} \sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2}$$

$$\sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

C

(28)

$$Z = -(\sqrt{2} - 1) \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$Z = e^{\pi i} (\sqrt{2} - 1) e^{\frac{\pi}{3}i} = (\sqrt{2} - 1) e^{\frac{4\pi}{3}i}$$

D

(29)

$$\begin{aligned} Z &= 2i \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \\ &= 2e^{\frac{\pi}{2}i} \cdot e^{\frac{\pi}{3}i} = 2e^{\frac{5\pi}{6}i} \end{aligned}$$

A

(30)

$$\begin{aligned} Z &= -2 \left[ \sin \frac{\pi}{3} + i \cos \frac{\pi}{3} \right] \\ &= -2 \left[ \cos \left( \frac{\pi}{2} - \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{2} - \frac{\pi}{3} \right) \right] \end{aligned}$$

$$= -2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$= e^{\pi i} \cdot 2 \cdot e^{\frac{\pi}{6}i} = 2e^{\frac{7\pi}{6}i}$$

A



(36)

$$\begin{aligned}
 p, q \in R \\
 Z_2 = 1 - i \\
 Z_1 + Z_2 = 2 \\
 Z_1 \cdot Z_2 = 1 + 1 = 2
 \end{aligned}$$

B

المعادلة التي جذراها  $Z_2, Z_1$  هي من الشكل:

$$\begin{aligned}
 Z^2 - 2Z + 2 = 0 \quad (p = -2, q = 2) \\
 \Rightarrow Z^2 - 2Z + 2 = 0 \\
 Z^2 + pZ + q = 0
 \end{aligned}$$

(37)

$$\begin{aligned}
 Z = e^{i\theta} \cdot e^{-i\theta} + e^{i\theta} \cdot e^{i\theta} \\
 Z = (e^{i\theta} + e^{-i\theta})e^{i\theta} = 2\cos\theta e^{i\theta}
 \end{aligned}$$

B

(38)

بفرض  $Z = x + iy$ 

$$\begin{aligned}
 (2x)^2 - (2iy)^2 = 4 \\
 4x^2 + 4y^2 = 4 \Rightarrow x^2 + y^2 = 1 \\
 \text{وهي معادلة دائرة مركزها } (0, 0) \text{ و } R = 1
 \end{aligned}$$

C

(39)

$$e^{i\frac{\theta}{2}} - e^{-i\frac{\theta}{2}} = 2i\sin\frac{\theta}{2}$$

C

حسب أويلر

(40)

$$\begin{aligned}
 \frac{1}{Z} &= \frac{1}{a + ib} = \frac{a - ib}{a^2 + b^2} \\
 &= \frac{a}{a^2 + b^2} + i \frac{-b}{a^2 + b^2}
 \end{aligned}$$

A

