(1)

$$ln\left|\frac{x-1}{x}\right|$$

$$D=\mathbb{R}\backslash\{0,1\}$$

B

(2)

$$f(x)=\frac{1}{x}+lnx$$

$$D_f=]0,+\infty[=\mathbb{R}_+^*$$

$$D_{f'}=\mathbb{R}_+^*$$

❖ التابع اللوغارتمي اشتقاقي على مجموعة تعريفه.

A

(3)

$$ln(x^2)=2ln|x|$$

$$lnx^2=2lnx\ ;x>0$$

$$lnx^2=2ln-x\ ;x<0$$

D

(4)

$$ln\sqrt{2x-3}=ln(6-x)-\frac{1}{2}lnx$$

$$D_1=\left]\frac{3}{2},+\infty\right[$$

$$D_2=]-\infty,6[$$

$$D_3=]0,+\infty[$$

$$D=\left]\frac{3}{2},6\right[$$

$$ln\sqrt{2x-3}=ln(6-x)-ln\sqrt{x}$$

$$ln\sqrt{2x-3}=ln\left(\frac{6-x}{\sqrt{x}}\right)$$

$$\sqrt{2x-3}=\frac{6-x}{\sqrt{x}}$$

نربع الطرفين:

$$2x-3=\frac{x^2-12x+36}{x}$$

$$2x^2-3x=x^2-12x+36$$

$$x^2+9x-36=0$$

$x=-12$ مرفوض

$x=3$ مقبول

B

(5)

$$\lim_{x\to+\infty}\frac{ln(x+1)}{x}$$

1) أوبيتال:

$$f(x)=\frac{ln(x+1)}{x}$$

$$\lim_{x\to+\infty}\frac{1}{x+1}=0$$

2) نضع $x=\frac{1}{X}$

$$x\longrightarrow+\infty\Longrightarrow X\longrightarrow0^+$$

$$f(X)=\frac{ln\left(\frac{1}{x}+1\right)}{\frac{1}{X}}=Xln\left(\frac{1+X}{X}\right)$$

$$=Xln(1+X)-Xln(X)$$

$$\lim_{x\to0}Xln(1+X)-Xln(X)=0$$

B

(6)

$$f(x)=\frac{lnx}{x^2}$$

$$D=]0,+\infty[$$

$$f'(x)=\frac{x-2xlnx}{x^4}=\frac{1-2lnx}{x^3}$$

مماس موازٍ لـ $y=x$

$$\Rightarrow f'(x)=1$$

$$\frac{1-2lnx}{x^3}=1$$

$$1-2lnx=x^3$$

$$g(x)=x^3-1+2lnx$$

$$g'(x)=3x^2+\frac{2}{x}$$

$$\frac{3x^3+2>0}{x>0}$$

x	0	(1)	$+\infty$
g'	$\|\|$	$+$	
g	$\|\|$	$-\infty \nearrow 0 \nearrow +\infty$	

للمعادلة $g(x)=0$ حل وحيد

A

$$\Rightarrow \frac{1-2lnx}{x^3} = 1$$

لها حل وحيد ⟸ مماس وحيد

(7)

$$ln(ln(x))$$

معرَّف من أجل:

B

$$ln(x) > 0$$
$$ln(x) > ln(1)$$
$$x > 1$$
$$D =]1, +\infty[$$

(8)

$$f(x) = 5 - 2x + 3ln\left(\frac{x+1}{x-4}\right)$$

$$D_f =]-\infty, -1[\cup]4, +\infty[$$

$$\lim_{x\to-\infty} f(x) = +\infty$$
$$\lim_{x\to-1^-} f(x) = -\infty$$
$$\lim_{x\to4^+} f(x) = +\infty$$
$$\lim_{x\to+\infty} f(x) = -\infty$$

$$f'(x) = -2 - \frac{15}{(x-4)(x+1)} < 0$$

x	$-\infty$ (1)	-1	4	(1) $+\infty$
f'	$-$			$-$
f	$+\infty$ ↘ $-\infty$			$+\infty$ ↘ $-\infty$

$$f(5) = -5 + 3ln6$$
$$= -5 + 3ln3 + 3ln2$$
$$= -5 + 5.4 = 0.4$$

D

$$f(6) = -7 + 3ln\left(\frac{7}{2}\right) < 0$$

$$x \in]5, 6[$$

(9)

$$f(x) = lnx$$

$$f'(x) = \frac{1}{x}, f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = +\frac{2}{x^3}, f^4(x) = \frac{-6}{x^4}$$

D

(10)

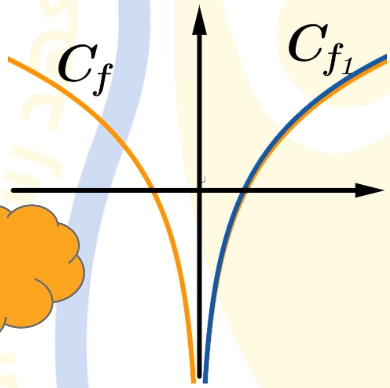
$$f(x) = ln(2x-1)$$
$$f(1) = 0$$

D

$$f'(x) = \frac{2}{2x-1}$$
$$f'(1) = 2$$

(11)

$$f(x) = lnx^2$$
$$f_1(x) = 2lnx$$
$$f(x) = f_1(x); x > 0$$



A

(12)

$$f(x) = \frac{x}{lnx} - e$$

$$f_1(x) = \frac{x}{ln(-x)} + e$$

$$D_f =]0, +\infty[\backslash\{1\}$$

C

$$D_{f_1} =]-\infty, 0[\backslash\{-1\}$$

$$f_1(x) = -f(-x)$$

C_1 نظير C بالنسبة للمبدأ

(13)

$$f(x) = ln(ax+b)$$

مقارب $x = \frac{1}{2}$

$$\Rightarrow \frac{a}{2} + b = 0 \ldots (1)$$

C يمر بـ $A(1,0)$

$$\Rightarrow ln(a+b) = 0$$
$$\Rightarrow a + b = 1 \ldots (2)$$

من (2):

$$b = 1 - a \ldots (3)$$

نعوض (3) في (1):

$$\frac{a}{2}+1-a=0$$

$$\Rightarrow a=2$$

D

بالتعويض في (3):

$$b=-1$$

(14)

$$f(x)=ln\left(\frac{2x+a-2}{ax+1}\right)$$

C يمر بـ $A(2,ln2)$

$$\Rightarrow ln\left(\frac{a+2}{2a+1}\right)=ln(2)$$

$$\frac{a+2}{2a+1}=2$$

$$a+2=4a+2$$

$$\Rightarrow a=0$$

A

(15)

$$f(x)=ln(x^2+a)$$

C يمر بـ $(2,ln3)$

$$\Rightarrow ln(4+a)=ln3$$

$$\Rightarrow a=-1$$

D

(16)

$$f(x)=ln\left(\frac{x}{2-x}\right)$$

$$x=1$$

$$f(1)=0$$

$$f'(x)=\frac{2}{x(2-x)}$$

$$f'(1)=2$$

$$y-y_0=m(x-x_0)$$

$$y-0=2(x-1)$$

$$y=2x-2$$

B

(17)

$$f(x)=\frac{lnx}{x}$$

$$f_1(x)=\frac{1}{x}ln\left(\frac{1}{x}\right)$$

$$ln\left(\frac{1}{x}\right)=-lnx$$

$$\Rightarrow f_1(x)=-\frac{lnx}{x}=-f(x)$$

A

(18)

$$f(x)=\frac{lnx}{x}$$

$$D_f=]0,+\infty[$$

$$\lim_{x\to 0} f(x)=-\infty$$

$$\lim_{x\to +\infty} f(x)=0$$

$$f'(x)=\frac{1-lnx}{x^2}$$

$$f'=0\Rightarrow x=e$$

$$f(e)=\frac{1}{e}$$

x	0		e	π	$+\infty$
f'	$\parallel$	$+$	0	$-$	
f	$\parallel$	$-\infty \nearrow$	$\frac{1}{e}$	$\searrow$	0

$$x\in]e,\pi[$$

C

(19)

$$f(x)=ln(x+2)$$

$$\Delta: y-x=0$$

$$\Rightarrow y=x$$

$$T\parallel \Delta_1\Rightarrow m=1$$

$$m=f'(a)=1$$

$$f'(x)=\frac{1}{x+2}$$

$$f'(a)=\frac{1}{a+2}=1$$

$$1=a+2\Rightarrow a=-1$$

$$f(-1)=0$$

C

(20)

$$ln(2.2)$$

ليكن: $f(x)=lnx$

$$2.2=2+0.2$$

(23)

$$f(x) = ln\left(\frac{x+1}{x-lnx}\right)$$
$$D_{lnx} =]0,+\infty[$$
$$x+1>0\ ;x\in]0,+\infty[$$
لیكن: $h(x) = x - lnx$
$$\lim_{x\to 0} h(x) = +\infty$$
$$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty}\left(1-\frac{lnx}{x}\right) = +\infty$$
$$h'(x) = 1-\frac{1}{x} = \frac{x-1}{x}$$
$$h' = 0 \Rightarrow x = 1$$

x	0		1		$+\infty$
f'	||	$-$	0	$+$	
f	|| $+\infty$	↘	1	↗	$+\infty$

$$x-lnx>0\ ;x\in]0,+\infty[$$
$$\Rightarrow D_f =]0,+\infty[$$

A

(24)

$$f(x) = \begin{cases}\frac{x}{x-lnx}, & x>0\\ 0 & ,\ x=0\end{cases}$$
$$f'(0) = \lim_{x\to 0} t(x)$$
$$t(x) = \frac{\frac{x}{x-lnx}}{x} = \frac{1}{x-lnx}$$
$$\lim_{x\to 0} t(x) = 0$$

C

(25)

$$f(x) = ln(ax)+bx$$
$$f(1) = 0 \Rightarrow ln(a)+b = 0 \ldots (1)$$
$$f'(1) = 0 \Rightarrow 1+b = 0 \ldots (2)$$
$$\Rightarrow b = -1 \ldots (3)$$
نعوض (3) في (1):
$$ln(a) = 1$$
$$\Rightarrow a = e$$

B

$$f(2) = 0.7$$
$$f'(x) = \frac{1}{x}$$
$$f'(2) = \frac{1}{2}$$
$$f(2.2) = 0.7+\frac{1}{2}\times 0.2$$
$$= 0.8$$

D

(21)

$$f(x) = \frac{x^2+x-lnx}{x}$$
$$f(x) = x+1-\frac{lnx}{x}$$
$$\Rightarrow y = x+1$$

B

(22)

$$f(x) = \frac{1}{x}lnx$$
$$D_f =]0,+\infty[$$
$$\lim_{x\to 0} f(x) = -\infty$$
$$\lim_{x\to+\infty} f(x) = 0$$
$$f'(x) = \frac{1-lnx}{x^2}$$
$$f' = 0 \Rightarrow x = e$$
$$f(e) = \frac{1}{e}$$

x	0		e		$+\infty$
f'	||	$+$	0	$-$	
f	|| $-\infty$	↗	$\frac{1}{e}$	↘	0

$$\pi > e$$
نصوّر الطرفين، وبما أن f متناقص على المجال: $[e,\pi]$، نقلب التراجح:
$$f(\pi) < f(e)$$
$$\frac{ln\pi}{\pi} < \frac{lne}{e}$$
نضرب الطرفين بـ $e\pi$:
$$eln\pi < \pi lne$$
$$ln\pi^e < lne^\pi$$
$$\pi^e < e^\pi$$

C

(26)

$$lnx - lny = ln3 \;...\;(1)$$
$$ln(x-y) = 2ln2 \;...\;(2)$$

من (1) نجد:

$$ln\frac{x}{y} = ln3$$
$$\frac{x}{y} = 3 \Rightarrow x = 3y \;...\;(3)$$

من (2) نجد:

$$x - y = 4$$

نعوض (3) في: $x - y = 4$

$$3y - y = 4 \Rightarrow y = 2$$

من (3): $x = 6$

D

(27)

$$f(x) = 1 + \frac{2lnx}{x}$$
$$x = 1.1$$
$$1.1 = 1 + 0.1$$
$$f(1) = 1$$
$$f'(x) = \frac{2-2lnx}{x^2} \Rightarrow f'(1) = 2$$
$$f(1.1) = 1 + 2 \times 0.1 = 1.2$$

B

(28)

$$z = ln5 \,, y = ln3 \,, x = ln2$$
$$ln\left(\frac{18}{20}\right) = ln\left(\frac{9}{10}\right)$$
$$= ln9 - ln10$$
$$= 2ln3 - ln5 - ln2$$
$$\Rightarrow ln\left(\frac{18}{20}\right) = 2y - z - x$$

A

(29)

$$\frac{f(x) + f(1-x)}{2} = -\frac{1}{4}$$

قانون مركز التناظر:

$$f(x) + f(2x_0 - x) = 2y_0$$
$$\frac{f(x) + f(2x_0 - x)}{2} = y_0$$

بالمطابقة:

$$2x_0 = 1 \Rightarrow x_0 = \frac{1}{2}$$
$$y_0 = -\frac{1}{4}$$

C

(30)

$$lny - 2lnx = 0$$
$$lny = 2lnx$$
$$y > 0 \,, x > 0$$
$$lny = lnx^2$$
$$y = x^2$$

B

(31)

$$f(x) = ln\big(E(x)\big)$$

معرَّف من أجل:

$$E(x) > 0 \Rightarrow x \geq 1$$

D

(32)

$$f'(x) = lnx$$

D

(33)

$$ln(x.y) = 1 \;...\;(1)$$
$$(lnx)(lny) = -12 \;...\;(2)$$

نضع:

$$lnx = X \,, lny = Y$$

من (1) نجد:

$$lnx + lny = 1$$
$$X + Y = 1 \Rightarrow X = 1 - Y \;...\;(3)$$

من (2) نجد:

$$X.Y = -12$$

نعوض (3) في (2):

$$(1-Y)Y=-12$$
$$-Y^2+Y=-12$$
$$Y^2-Y-12=0$$
$$Y=+4 \Longrightarrow X=-3$$
$$Y=-3 \Longrightarrow X=+4$$
$$Y=4 \Longrightarrow y=e^4$$
$$X=-3 \Longrightarrow x=\frac{1}{e^3}$$

A

(34)

$$x^2-2x+ln(m+1)=0$$

معادلة لها جذر مضاعف

$$\Longrightarrow \Delta=0\,,m>-1$$
$$\Delta=4-4ln(m+1)=0$$
$$4=4ln(m+1)$$
$$ln(m+1)=1$$
$$m+1=e$$
$$\Longrightarrow m=e-1$$

C

(35)

$$f(x)=\frac{1}{x}+lnx$$
$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{1}{x}\left(1+xln(x)\right)=+\infty$$
$$\lim_{x\to +\infty} f(x)=+\infty$$
$$f'(x)=-\frac{1}{x^2}+\frac{1}{x}=\frac{-1+x}{x^2}$$
$$f'=0 \Longrightarrow x=1$$
$$f(1)=1$$

x	0		1		$+\infty$
f'	$\|\|$	$-$	0	$+$	
f	$\|\|$ $+\infty$	$\searrow$	1	$\nearrow$	$+\infty$

A

(36)

$$f(x)=ln\left(\frac{x-2}{x+2}\right)$$
$$D_f=]-\infty,-2[\cup]2,+\infty[$$

يرسم بفرعين C_f

$$C_{f_1}\in]-\infty,-2[$$
$$C_{f_2}\in]2,+\infty[$$
$$f(x)+f(2(0)-x)=2(0)$$

D

(37)

$$\lim_{x\to 0}(x+x(lnx)^2)$$
$$f(x)=x+x(lnx)^2$$
$$=x+x(ln\sqrt{x})^2$$
$$=x+4(\sqrt{x}ln\sqrt{x})^2$$
$$\lim_{x\to 0} f(x)=0$$

A

(38)

$$v_n=8.\left(\frac{1}{2}\right)^n \quad ,w_n=ln(v_n)$$
$$v_0=8 \Longrightarrow w_0=2.1$$
$$v_1=4 \Longrightarrow w_1=1.4$$
$$v_2=2 \Longrightarrow w_2=0.7$$

B

(39)

$$f(x)=\begin{cases} x^2(1-lnx), & x>0 \\ 1 \quad , & x=0 \end{cases}$$
$$t(x)=\frac{x^2(1-lnx)-1}{x}$$
$$=x-xlnx-\frac{1}{x}$$
$$\lim_{x\to 0} t(x)=-\infty$$

C

(40)

$$f(x)=x-xln\left(1+\frac{1}{x}\right)$$
$$y=x-1$$
$$f-y=-xln\left(1+\frac{1}{x}\right)+1$$

نضع: $x=\frac{1}{X} \Longrightarrow X=\frac{1}{x}$

$$x\longrightarrow +\infty \Longrightarrow X\longrightarrow 0^+$$
$$f-y=-\frac{ln(1+x)}{x}+1$$
$$\lim_{x\to 0} f-y=-1+1=0$$

B