

STATISTICS

FINAL REVISION



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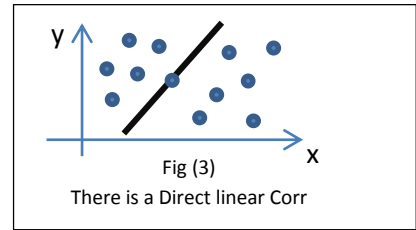
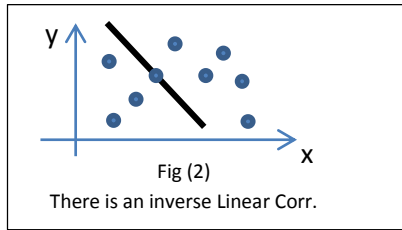
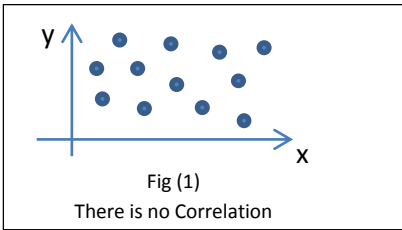
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Chapter One :-

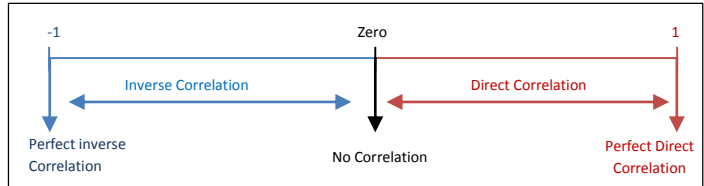
First : Correlation

Types of Correlation :-



*Important Notes

خلي بالك لو قالك **Strongest/Weakest** و مقالش انه **Inverse** يبقى اكيد **Direct** ... هو عمره م هيقول **Direct** لان ده الطبيعي .



Example :- → (-0.8, -0.5, 0.4, 0.7)

مش هقول **Direct**
بس اكيد **Direct**

- Strongest correlation coefficient is 0.7
- Weakest correlation coefficient is 0.4
- Strongest Inverse correlation coeff. Is -0.8
- Weakest Inverse correlation coeff. Is -0.5

Pearson's Correlation Coefficients :

$$r = \frac{n \sum xy - \sum x \times \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \times \sqrt{n \sum y^2 - (\sum y)^2}}$$

Spearman's Rank Correlation Coefficients :

$$r = 1 - \frac{6 \sum D^2}{n(n^2 - 1)}$$

1 The next table shows the marks of 10 students in the history and geography subjects.

| | | | | | | | | | | |
|-------------|----|----|----|----|----|----|----|----|----|----|
| History x | 75 | 80 | 93 | 65 | 87 | 71 | 98 | 69 | 84 | 78 |
| Geography y | 82 | 78 | 86 | 72 | 91 | 80 | 95 | 73 | 89 | 74 |

Required is to calculate the Pearson's Correlation Coefficient between the two variables x and y and identify the type of the Correlation .

Solution

We Form the next Table

| x | y | x ² | y ² | xy |
|----------|----------|----------------|----------------|-----------|
| 75 | 82 | 5625 | 6724 | 6150 |
| 80 | 78 | 6400 | 6084 | 6240 |
| 93 | 86 | 8649 | 7396 | 7998 |
| 65 | 72 | 4225 | 5184 | 4680 |
| 87 | 91 | 7569 | 8281 | 7917 |
| 71 | 80 | 5041 | 6400 | 5680 |
| 98 | 95 | 9604 | 9025 | 9310 |
| 69 | 73 | 4761 | 5329 | 5037 |
| 84 | 89 | 7056 | 7921 | 7476 |
| 78 | 74 | 6084 | 5476 | 5772 |
| $\sum x$ | $\sum y$ | $\sum x^2$ | $\sum y^2$ | $\sum xy$ |
| = 800 | = 820 | =65014 | =67820 | = 66260 |

$$\therefore r = \frac{n \sum xy - (\sum x \times \sum y)}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\therefore r = \frac{10 \times 66260 - (800 \times 820)}{\sqrt{10 \times 65014 - (800)^2} \sqrt{10 \times 67820 - (820)^2}}$$

$$= \frac{6006}{\sqrt{10140} \sqrt{5800}} \approx 0.8608 \quad \text{The Correlation is Direct}$$

2

Find the Pearson's correlation coefficient between the two variables x and y and identify its type if :-

$$\sum x = 68 \quad , \quad \sum y = 36 \quad , \quad \sum xy = 348$$

$$\sum x^2 = 620 \quad , \quad \sum y^2 = 204 \quad , \quad n = 8$$

Solution

$$\therefore r = \frac{n \sum xy - (\sum x \times \sum y)}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\therefore r = \frac{8 \times 348 - (68 \times 36)}{\sqrt{8 \times 620 - (68)^2} \sqrt{8 \times 204 - (36)^2}} = \frac{336}{\sqrt{336} \sqrt{336}} = 1$$

The value of the correlation coefficient (+1) means that the relation between the two variables x and y is perfect direct .

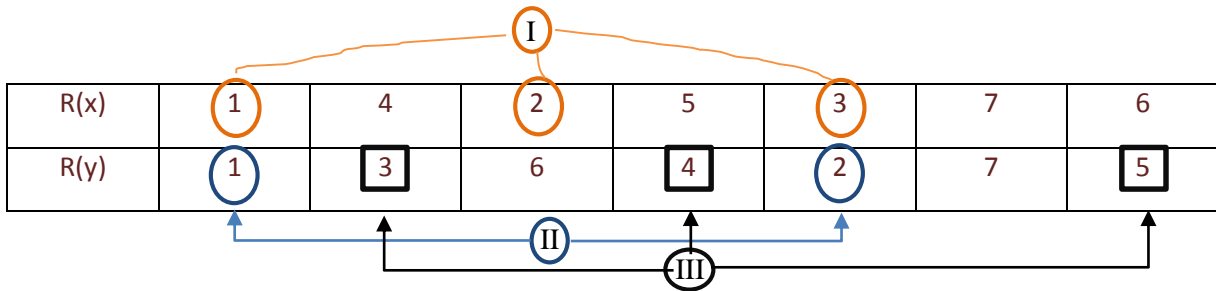
3

Find the Spearman's rank correlation coefficient and identify its type

| | | | | | | | |
|-----------|------|------|------|------|------|-----------|-----------|
| Subject 1 | Weak | Pass | Weak | Good | Weak | Excellent | Very good |
| Subject 2 | Weak | Pass | Good | Pass | Weak | Very good | pass |

Solution

* أولاً : هنرتب كل صف من الصغير للكبير .. و سيبك من التكرار , رتب عادي و بعدين هنحل مشكله التكرار " في الاخر " .



* ثانياً : نحل مشكله التكرار ب اننا هنكتب لكل المتكرر نفس الرقم و هو مجموعهم عددهم .

$$\textcircled{I} \frac{1+2+3}{3} = 2$$

$$\textcircled{II} \frac{1+2}{2} = 1.5$$

$$\textcircled{III} \frac{3+4+5}{3} = 4$$

عددهم

* ثالثاً : نظبط الجدول و نعمل " D = R(x) - R(y) "

| R(x) | R(y) | D | D ² |
|------|------|-----|-------------------|
| 2 | 1.5 | 0.5 | 0.25 |
| 4 | 4 | 0 | 0 |
| 2 | 6 | -4 | 16 |
| 5 | 4 | 1 | 1 |
| 2 | 1.5 | 0.5 | 0.25 |
| 7 | 7 | 0 | 0 |
| 6 | 4 | 2 | 4 |
| | | | $\sum D^2 = 21.5$ |

N = 7 "عددهم"

$$\therefore r = 1 - \frac{6 \sum D^2}{n (n^2 - 1)}$$

$$= 1 - \frac{6 \times 21.5}{7 (49 - 1)}$$

= 0.6161 It's Direct Correlation

4

Choose the correct answer :

1) The strongest correlation coefficient of the following is :

- a) - 0.94 b) Zero c) 0.5 d) 0.85

2) The strongest inverse correlation coefficient of the following is :

- a) - 0.2 b) - 0.5 c) - 0.7 d) - 0.8

3) The scatter diagram representing inverse correlation is :



4) The weakest correlation coefficient of the following is :

- a) - 1.2 b) - 0.7 c) 0.12 d) 0.9

5) One of the next numbers can represent the strongest inverse correlation coefficient between two variables :

- a) 0.3 b) 0.9 c) - 1.1 d) - 0.95

Solution

1)

" هنا هو مقالش **Inverse** ف كده هو يقصد **Direct** " هختار اقرب رقم ل -1

d) 0.85

2)

"عايز أقوى **Inverse** ف كده هو يقصد أقرب رقم لل -1"

d) - 0.8

3) b)

4)

"عايز أضعف يعني أقرب رقم لل 0"

e) 0.12 "لازم +ve طبعاً"

5) (0.3 , 0.9)

Not Inverse

(-1.1)

Not Correlation

$-1 \leq r \leq 1$

∴ d) - 0.95

5

Calculate the Spearman's rank correlation between x and y through the data of the next table :

| | | | | | | |
|---|---|---|---|---|---|----|
| X | 4 | 7 | 8 | 5 | 8 | 12 |
| y | 7 | 6 | 6 | 4 | 6 | 10 |

Solution

هنرتب الأرقام من الصغير للكبير و نعمل نفس الكلام زي رقم 3

From the next Table:

| X | Y | Ranks x | Ranks y | D | D ² |
|----|----|---------|---------|-----|----------------|
| 4 | 7 | 1 | 5 | -4 | 16 |
| 7 | 6 | 3 | 3 | 0 | 0 |
| 8 | 6 | 4.5 | 3 | 1.5 | 2.25 |
| 5 | 4 | 2 | 1 | 1 | 1 |
| 8 | 6 | 4.5 | 3 | 1.5 | 2.25 |
| 12 | 10 | 6 | 6 | 0 | 0 |

$$\sum D^2 = 21.5$$

$$\therefore r = 1 - \frac{6 \sum D^2}{n(n^2-1)}$$

$$\therefore r = \frac{6 \times 21.5}{6(36-1)} \approx 0.3857$$

The correlation is direct

Second : Regression

$$y = a + bx$$

$$a = \frac{\sum y - b \times \sum x}{n}$$

$$b = \frac{n \sum xy - \sum x \times \sum y}{n \sum x^2 - (\sum x)^2}$$

Error = |table value - Equation value|

1 The next table illustrates the production of a summer crop (y) of the cultivated land (x) in Feddan

| | | | | | | | | | | |
|-------------------------------|-----|-----|-----|-----|-----|-------|-------|------|------|------|
| Cultivated land (x) in feddan | 50 | 200 | 110 | 80 | 120 | 74.5 | 88.9 | 5.7 | 11 | 3.2 |
| Production of (y) in kg | 140 | 500 | 400 | 300 | 356 | 240.5 | 200.6 | 33.5 | 69.8 | 18.7 |

First : find the regression line equation.

Second : predict the value of the production in kg if the cultivated land is 100 feddans.

Third : Find the error in the production if you know that the cultivated land is 120 feddans.

Solution

| X | y | x ² | y ² | Xy |
|------|-------|----------------|----------------|-----------|
| 50 | 140 | 2,500 | 19,600 | 7,000 |
| 200 | 500 | 40,000 | 250,000 | 100,000 |
| 110 | 400 | 12,100 | 160,000 | 44,000 |
| 80 | 300 | 6,400 | 90,000 | 24,000 |
| 120 | 356 | 14,400 | 126,736 | 42,720 |
| 74.5 | 240.5 | 5,550.25 | 57,840.25 | 17,917.25 |
| 88.9 | 200.6 | 7,903.21 | 40,240.36 | 17,833.34 |
| 5.7 | 33.5 | 32.49 | 1,122.25 | 190.95 |
| 11 | 69.8 | 121 | 4,872.04 | 767.8 |
| 3.2 | 18.7 | 10.24 | 349.69 | 59.84 |

First :-

$\therefore \sum x = 743.3$, $\sum y = 2259.1$

$\sum x^2 = 89017.19$, $\sum xy = 254489.18$

$\therefore b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{10 \times (254489.18) - (743.3) \times (2259.1)}{10 \times (89017.19) - (743.3)^2}$

$= 2.5637$

$a = \frac{\sum y - b \sum x}{n} = \frac{2259.1 - 2.5637 \times 743.3}{10}$

$= 35.35$

\therefore The regression line equation is $y = 35.35 + 2.5637x$

Second : $y = ??$, $x = 100$

By substitute at the equation $y = 35.35 + 2.5637 \times 100$
 $= 291.72 \text{ Kg}$

Third : Error = |Table value - eq. value| at $x=120$

$y = 35.35 + 2.5637 \times 120 = 343$ "eq. value"

$y = 356$ "table value"

error = | 356 - 343 | = 13

2 Mining the following table shows the data about the average price of the oil barrel and the rates of the economic growth in a country within eight years . Required is to :

| | | | | | | | | |
|----------------------------------|------|-----|------|------|------|------|------|------|
| Price of the oil barrel (x) | 36 | 40 | 36.2 | 31.1 | 29.7 | 16.3 | 18.7 | 14.6 |
| Rates of the economic growth (y) | 0.91 | 3.5 | 3.2 | 2.7 | 2.3 | -1 | -0.9 | -1.6 |

First : graph the scatter diagram and show the type of the correlation

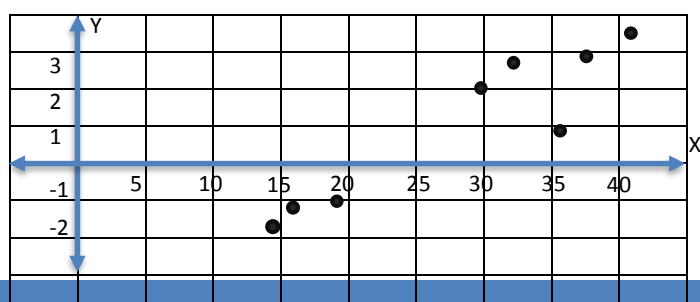
Second : Find the regression line equation of the given data

Third : Predict the economic growth when the price of the oil barrel is \$15 , then when the price is \$35

Solution

First

The opposite figure represents the scatter diagram and it shows also the correlation is direct .



| X | Y | x^2 | y^2 | X y |
|--------------|-------------|----------------|----------------|---------------|
| 36 | 0.91 | 1296 | 0.8281 | 32.76 |
| 40 | 3.5 | 1600 | 12.25 | 140 |
| 36.2 | 3.2 | 1310.44 | 10.24 | 115.84 |
| 31.1 | 2.7 | 967.21 | 7.29 | 83.97 |
| 29.7 | 2.3 | 882.09 | 5.29 | 68.31 |
| 16.3 | -1 | 265.69 | 1 | -16.3 |
| 18.7 | -0.9 | 349.69 | 0.81 | -16.83 |
| 14.6 | -1.6 | 213.16 | 2.56 | -23.36 |
| 222.6 | 9.11 | 6884.28 | 40.2681 | 384.39 |

Second : we calculate the value of the constant b from the relation :

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{8 \times 384.39 - (222.6 \times 9.11)}{8 \times 6884.28 - (222.6)^2} \approx 0.1896$$

$$\therefore a = \frac{\sum y - b \sum x}{n} \therefore a = \frac{9.11 - (0.1896 \times 222.6)}{8} \approx -4.1368$$

\therefore The regression line equation is : $\hat{y} = a + bx$

$$Y = -4.1368 + 0.1896x$$

From the table data:

$$\sum y = 9.11, \quad \sum x = 222.6, \quad \sum xy = 384.39, \\ \sum x^2 = 6884.28, \quad \sum y^2 = 40.2681$$

Third

When $x = 15$, then: $y = 0.1896 \times 15 - 4.1368 \approx -1.2928$

When $x = 35$, then: $y = 0.1896 \times 35 - 4.1368 \approx 2.4992$

3

Choose the correct answer:

1) the statistical equation of the regression line equation where b is the regression coefficient is :

a) $\hat{y} = ax + b$ b) $\hat{y} = a + bx$ c) $\hat{y} = ay + b$ d) $\hat{y} = a + by$

2) if the regression line equation is : $\hat{y} = 2 + 0.5x$, then the value of y expected when $x = 6$ is :

a) 4 b) 5 c) 7 d) 8

3) if the two points (11.5 , 10) and (6.5 , 5) lie on the regression line y on x, then the correlation between x and y is :

a) Direct b) Inverse c) Perfect d) Nihilistic

4) if the two points (5 , 13) and (14 , 4) lie on the regression line y on x, then all the following points lie on the same line except :

a) (15 , 5) b) (10 , 8) c) (6 , 12) d) (5 , 13)

5) if all the points in a scatter diagram lie on straight line whose slope is negative, then the correlation coefficient between the two variables equals :

a) 1 b) zero c) -0.5 d) -1

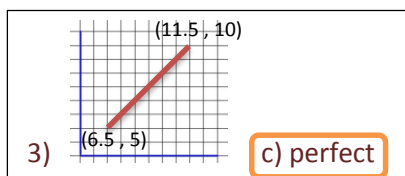
6) if all the points in a scatter diagram lie on a straight line whose slope is positive, then the correlation coefficient between the two variables equals :

a) -1 b) Zero c) $\frac{1}{2}$ d) 1

Solution

1) **b) $y = a + bx$**

2) $y = 2 + 0.5 \times 6$
= **b) 5**



4) $m = \frac{4-13}{14-5} = -1$

a) $\frac{4-5}{14-15} = 1$

b) $\frac{4-8}{14-10} = -1$

c) $\frac{4-12}{14-6} = -1$

d) $\frac{4-13}{14-5} = -1$

→ a) (15 , 5)

5) **d) -1**

"طالما Line واحد تبقي"

"Perfect"

"زي 3"

6) **d) 1**

Chapter 2 :

Rules Of Probability

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

3) $P(\bar{A}) = 1 - P(A)$

4) $P(A - B) = P(A) - P(A \cap B)$

5) $P(A' \cup B') = 1 - P(A \cap B)$

Or $P(A \cap \bar{B})$

6) $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

7) $P(\bar{B} \cap A) = P(A \cap B')$ "Important"

Mutually Exclusive Events

$P(A \cap B) = 0$

$A \subset B \begin{cases} 1) P(A \cap B) = P(A) \\ 2) P(A \cup B) = P(B) \end{cases}$

Two Independent Events :-

$P(A \cap B) = P(A) \times P(B)$

Conditional Probability :

$P(A|B) = \frac{P(A \cap B)}{P(B)}$



Note : $P(A|B') = \frac{P(A \cap B')}{P(B')}$
 $= \frac{P(A) - P(A \cap B)}{1 - P(B)}$

Remember That :-

- 1) And $\rightarrow \times$
- 2) Or $\rightarrow +$

1

A regular die has been rolled once. Calculate the probability of appearing the number 2 known that the number appeared is even

Solution

Sample Space $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2\}$
 $B = \{2, 4, 6\}$ Then, $P(B) = \frac{3}{6} = \frac{1}{2}$, $P(A \cap B) = P(A) = \frac{1}{6}$
 $\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$
 $\therefore P(A|B) = \frac{1}{6} \div \frac{1}{2} = \frac{1}{6} \times 2 = \frac{1}{3}$

B: no. is even
A: no. is 2

2

If A and B are two events of the sample space where $P(A) = 0.45$, $P(B) = 0.6$, $P(B|A) = 0.8$ Find :

- a) $P(A \cap B)$ b) $P(A \cup B)$ c) $P(A|B)$ d) $P(\bar{B}|A)$

Solution

a) $\therefore P(B|A) = \frac{P(B \cap A)}{P(A)}$ "نستغلها الاول"
 $\therefore 0.8 = \frac{P(A \cap B)}{0.45}$
 $\therefore P(A \cap B) = 0.8 \times 0.45 = 0.36$
 b) $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.45 + 0.6 - 0.36 = 0.69$
 c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.36}{0.6} = 0.6$
 d) $P(\bar{B}|A) = \frac{P(B' \cap A)}{P(A)} = \frac{P(A - B)}{P(A)}$
 $= \frac{P(A) - P(A \cap B)}{P(A)} = \frac{0.45 - 0.36}{0.45} = 0.2$

3

From the data in the next table :

| Case | Number of people | |
|-------|------------------|---------------------|
| | Wear glasses | Do not wear glasses |
| Man | 800 | 600 |
| Woman | 400 | 200 |

Find the probability of a woman wearing glasses

Solution

"هو هنا طالب احتمال اني اختار Women و ده هعتبره A بشرط انها تكون لايبسه نظاره عموما و ده هعتبره B."

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

ركز هنا :
 يعني A, B الاثنين حصلوا مع بعض
 and
 $P(A \cap B) = \frac{400}{2000} = \frac{1}{5}$

"يعني B حصلت"
 $P(B) = \frac{1200}{2000} = \frac{3}{5}$
 هنا ال B هي انه يكون لايبس نظاره عموما سواء رجل او ست

$\therefore P(B) = \frac{1}{5} \div \frac{3}{5}$
 $= \frac{1}{3}$

4

A bag contains 10 white balls and 15 red balls. Two balls have been consecutively drawn without replacing. What is the probability the two drawn balls are white?

Solution

$$\frac{10}{25} \times \frac{9}{24} = \frac{3}{20}$$

5

100 learners are studying in a language institute. The number of learners studying English is 60, the number of the learners studying French is 50 and the number of the learners studying both languages is 35. If a learner has been randomly chosen from the institute, Find the probability the learner studies: -
 a) a language at least.
 b) English if he (she) studies French.
 c) French if he (she) studies English.

Solution

* $P(A) = \frac{60}{100} = 0.6$ "English"

$P(B) = \frac{50}{100} = 0.5$ "French"

$P(A \cap B) = \frac{35}{100} = 0.35$

a) $P(A \cup B) = ??$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.6 + 0.5 - 0.35$$

= 0.75

b) $P(A|B) = ??$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.35}{0.5}$$

= 0.7

c) $P(B|A) = ??$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.35}{0.6}$$

= 0.583

6

Choose the Correct answer :-

1) in an experiment for tossing a regular coin twice, the probability of appearing Tail in the second toss if Head appears in the first toss equals :

- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1

2) in an experiment for rolling a regular die once, the probability of appearing a prime even, number greater than 1 :

- a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

3) in an experiment for rolling a regular die once, the probability of appearing the number 3 known that the appearing number is odd is :

- a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

4) if $P(A \cap B) = \frac{2}{5}$, $P(A) = \frac{4}{5}$, then $P(B|A) =$

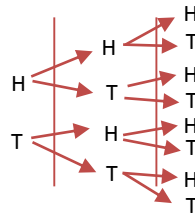
- a) $\frac{1}{2}$ b) $\frac{8}{28}$ c) $\frac{1}{4}$ d) $\frac{2}{5}$

5) if $P(A|B) = \frac{1}{3}$, $P(B) = \frac{12}{25}$ then $P(A \cap B) =$

- a) $\frac{4}{25}$ b) $\frac{1}{4}$ c) $\frac{25}{36}$ d) $\frac{16}{25}$

Solution

1)



- $S = \{ (H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T) \}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{2}{8}}{\frac{4}{8}} = \text{b) } \frac{1}{2}$$

4) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{\frac{2}{5}}{\frac{4}{5}} = \text{a) } \frac{1}{2}$$

2) $S = \{1, 2, 3, 4, 5, 6\}$

A : Prime Even
B : Greater Than 1

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{6}}{\frac{6}{6}} = \text{a) } \frac{1}{5}$$

3) $S = \{1, 2, 3, 4, 5, 6\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{6}}{\frac{3}{6}} = \text{b) } \frac{1}{3}$$

5) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

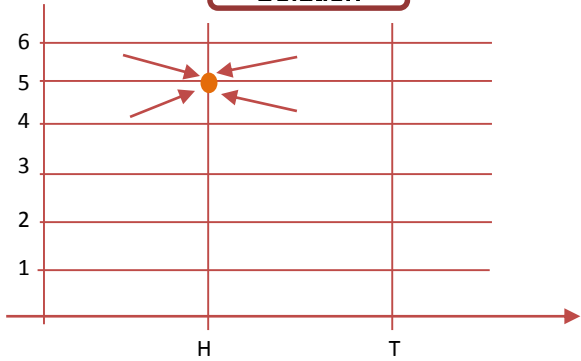
$$\rightarrow \frac{1}{3} = \frac{P(A \cap B)}{\frac{12}{25}}$$

$$P(A \cap B) = \frac{1}{3} \times \frac{12}{25}$$

a) $\frac{4}{25}$

7 What is the probability of appearing Head and number 5 in an experiment of tossing a coin once , then rolling a dice ?

Solution



$$P(A) = \frac{1}{12}$$

8 If A and B are two events of a sample space of a random experiment S and $P(A) = 0.5, P(B) = 0.6$ and $P(A \cup B) = 0.8$ Explain if A and B are two independent events.

Solution

$$\begin{aligned} \therefore P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ \therefore P(A \cap B) &= 0.5 + 0.6 - 0.8 = 0.3 \quad (1) \\ \therefore P(A) \times P(B) &= 0.5 \times 0.6 = 0.3 \quad (2) \end{aligned}$$

From (1) and (2), A and B are two independent events.

9 A man and his wife have insured their life at a life insurance company. If the company has estimated the probability that the man will live more than 20 years to be 0.2 and the probability that his wife will live more than 20 years to be 0.3 , Find the probability that : -
a) the man and his wife will live more than 20 years together
b) At least one of them will live more than 20 years
c) only one of them will live more than 20 years

Solution

واضح طبيعا ان A,B مستقلين

Let A be the event that the man will live more than 20 years
 $\therefore P(A) = 0.2$
B the event that the wife will live more than 20 years
 $\therefore P(B) = 0.3$

a)The probability that the man and his wife live more than 20 years together = $P(A \cap B)$
 $\therefore P(A \cap B) = P(A) \times P(B)$
 $\therefore P(A \cap B) = 0.2 \times 0.3 = 0.06$

b)The probability that at least one of them will live more than 20 years = $P(A \cup B)$
 $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $P(A \cup B) = 0.2 + 0.3 - 0.06 = 0.44$

c)The probability that only one of them will live more than 20 years = $P(A \cup B) - P(A \cap B)$
 $P(A \cup B) - P(A \cap B) = 0.44 - 0.06 = 0.38$

10 A bag contains 6 blue balls and 4 red balls . A ball is randomly drawn , then it is turned back to the bag , then another ball is drawn. What is the probability :
a) The two balls are red in the two times
b) The two balls are blue in the two times
c) The first ball is red and the second is blue
d) One ball is red and the other is blue

Solution

$$\begin{aligned} \text{a) } \frac{4}{10} \times \frac{4}{10} &= \frac{4}{25} \\ \text{b) } \frac{6}{10} \times \frac{6}{10} &= \frac{9}{25} \\ \text{c) } \frac{4}{10} \times \frac{6}{10} &= \frac{6}{25} \\ \text{d) } \frac{4}{10} \times \frac{6}{10} + \frac{6}{10} \times \frac{4}{10} &= \frac{12}{25} \end{aligned}$$

red and blue or blue and red

11 If S is the sample space of a random experiment where $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 4, 8\}$ and $B = \{2, 5, 6, 7\}$ Are A and B two independent events ? explain

Solution

$$\begin{aligned} \therefore P(A) &= \frac{4}{8} = \frac{1}{2} & \therefore P(B) &= \frac{4}{8} = \frac{1}{2} \\ \therefore A \cap B &= \{2\} & \therefore P(A \cap B) &= \frac{1}{8} \quad (1) \\ \therefore P(A) \times P(B) &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad (2) \end{aligned}$$

From (1) and (2), $P(A \cap B) \neq P(A) \times P(B)$
So, A and B are dependent events

12 A bag contains 6 blue balls and 4 red balls. If two balls are drawn one after another without replacing. What is the probability:
a) The two balls are red ?
b) The two balls are blue ?
c) The first ball is red and the second is blue ?

Solution

$$\begin{aligned} \text{a) } \frac{4}{10} \times \frac{3}{9} &= \frac{2}{15} \\ \text{b) } \frac{6}{10} \times \frac{5}{9} &= \frac{1}{3} \\ \text{c) } \frac{4}{10} \times \frac{6}{9} &= \frac{4}{15} \end{aligned}$$

13

Choose the correct answer :

- 1) If A and B are two independent events, $P(A) = 0.2, P(B) = 0.6$, then $P(A \cup B) =$
a) 0.12 b) 0.32 c) 0.68 d) 0.8
- 2) If A and B are two independent events $P(A) = 0.25, P(B) = 0.4$, then $P(A - B) =$
a) 0.1 b) 0.15 c) 0.3 d) 0.65
- 3) If A and B are two independent events $P(A) = 0.3, P(B) = x, P(A \cup B) = 0.72$ then x equals:
a) 0.24 b) 0.28 c) 0.4 d) 0.6

Solution

1) Independent $\rightarrow P(A \cap B) = P(A) \times P(B)$

$$\therefore P(A \cap B) = 0.2 \times 0.6 = \boxed{0.12}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.2 + 0.6 - 0.12 = \boxed{c) 0.68}$$

2) Independent $\rightarrow P(A \cap B) = P(A) \times P(B)$

$$\therefore P(A \cap B) = 0.25 \times 0.4 = \boxed{0.1}$$

$$P(A - B) = P(A) - P(A \cap B) = \boxed{c) 0.15}$$

3) $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore 0.72 = 0.3 + x - 0.3x$$

"Independent عشان"

$$\therefore 0.72 - 0.3 = 0.7x$$

$$\therefore x = \boxed{d) 0.6}$$

14

Choose the correct answer:

- 1) If A and B are two mutually exclusive events, $P(A) = 0.2$ and $P(B') = 0.6$, then $P(A \cup B) =$
a) 0.4 b) 0.6 c) 0.8 d) 1.2
- 2) If $A \subset B, P(A') = \frac{7}{10}$ and $P(B) = \frac{1}{2}$ then $P(A | B) =$
a) $\frac{1}{5}$ b) $\frac{3}{5}$ c) $\frac{4}{5}$ d) $\frac{3}{20}$
- 3) If A and B are two independent events, $P(A) = 0.2$ and $P(B) = 0.5$, then $P(A \cap B) =$
a) 0.7 b) 0.4 c) 0.3 d) 0.1
- 4) If $S = \{A, B, C\}$ and A, B and C are mutually exclusive events where $P(A) = 0.25$ and $P(B) = 0.4$, then $P(C) =$
a) 0.1 b) 0.15 c) 0.35 d) 0.65
- 5) If A and B are two independent events of S where $P(B) = 0.6$ and $P(A \cup B) = 0.68$, then $P(A) =$
a) 0.2 b) 0.3 c) 0.4 d) 0.5

Solution of no 14

1) Mutually Exclusive $\rightarrow P(A \cap B) = \emptyset$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.2 + 0.4 - 0$$

$$= \boxed{c) 0.8}$$

Note:

$$P(B') = 1 - P(B)$$

$$P(B) = 0.4$$

2) $A \subset B \left\{ \begin{array}{l} P(A \cup B) = P(B) \\ P(A \cap B) = P(A) \end{array} \right.$

$$\therefore P(A') = 1 - P(A) \quad \therefore P(A) = 1 - \frac{7}{10} = 0.3$$

$$P(A \cap B) = P(A) = 0.3$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{\frac{1}{2}} = 0.6 = \boxed{b) \frac{3}{5}}$$

3) Independent $\rightarrow P(A \cap B)$

$$= P(A) \times P(B)$$

$$\therefore P(A \cap B) = 0.2 \times 0.5 =$$

$$\boxed{d) 0.1}$$

4) $\therefore S = \{A, B, C\}$

$$\therefore P(A) + P(B) + P(C) = 1$$

$$\therefore P(C) = 1 - 0.4 - 0.25 =$$

$$\boxed{c) 0.35}$$

5) Let $P(A) = x$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore 0.68 = x + 0.6 - 0.6x$$

$$\therefore x = P(A) = \boxed{a) 0.2}$$

Chapter 3 :

First : Discrete Random Variable

$\sum F = 1$ "Important" Expectation (μ) = $\sum x \cdot f$ "Mean" Standard Deviation (σ) = $\sqrt{\sigma^2}$

Variance (σ^2) = $\sum x^2 \cdot f - (\mu)^2$

Coefficient of Variation = $(\frac{\sigma}{\mu} \times 100)\%$

Second : Continuous Random Variable

$f(x) = \begin{cases} \varphi, & a \leq x \leq b \\ 0, & \text{Otherwise} \end{cases}$

$p(L \leq x \leq \mu) = \frac{1}{2} [f(L) + f(\mu)] \times (\mu - L)$

$p(a \leq x \leq b) = 1$

$p(x \geq L) = p(L \leq x \leq b)$

"البدايه و النهايه الاساسيين"

"معايي البدايه L لكن مش معايي نهايه فهاخد النهايه الاساسيه"

$p(x \leq L) = p(a \leq x \leq L)$

"معايي النهايه L بس مش معايي البدايه فهاخد البدايه الاساسيه"

1 If x is a discrete random variable whose probability distribution is shown in the below table :

| | | | | | |
|----------|-----|-----|-----|---|-----|
| X_r | -1 | 0 | 1 | 2 | 3 |
| $F(x_r)$ | 0.3 | 0.1 | 0.1 | a | 0.2 |

First : find the value of a
Second : find the expectation (Mean)

Solution

First

$\therefore \sum f = 1$
 $\therefore 0.3 + 0.1 + 0.1 + a + 0.2 = 1$
 $\therefore a = 0.3$

Second

"علشان اجيب $\mu = \sum x \cdot f$ هجيب خانه لل $x \cdot f$ و اجمعها"

| | | | | | |
|-------------|------|---|-----|-----|-----|
| $X \cdot f$ | -0.3 | 0 | 0.1 | 0.6 | 0.6 |
|-------------|------|---|-----|-----|-----|

$\therefore \mu = \sum x \cdot f = -0.3 + 0.1 + 0.6 + 0.6 = 1$
"Mean"

2 If x is a discrete random variable whose probability distribution is shown as follows

| | | | | | |
|----------|-----|-----|-----|---|-----|
| X_r | 0 | 1 | 2 | b | 6 |
| $F(x_r)$ | 0.1 | 0.1 | 0.3 | a | 0.3 |

Find The value of a and b if the expectation $\mu = 3.5$

Solution

$\therefore \sum f = 1 \rightarrow 0.1 + 0.1 + 0.3 + a + 0.3 = 1$
 $\rightarrow a = 0.2$

| | | | | | |
|-------------|---|-----|-----|------|-----|
| $X \cdot f$ | 0 | 0.1 | 0.6 | 0.2b | 1.8 |
|-------------|---|-----|-----|------|-----|

$\therefore \mu = 3.5 \rightarrow \sum x \cdot f = 3.5$
 $\rightarrow 0.1 + 0.6 + 0.2b + 1.8 = 3.5$
 $\rightarrow b = 5$

3 If X is a discrete random variable whose probability distribution function is $f(x) = \frac{x+4}{16}$

where $x = -2, m, 1, 2$ Find the value of m, then find the mean and the variance of the discrete random variable X.

Solution

| x | f(x) | x . f | $x^2 \cdot f$ |
|----|----------------------------------|-----------------|-----------------|
| -2 | $\frac{-2+4}{16} = \frac{2}{16}$ | $-\frac{4}{16}$ | $\frac{8}{16}$ |
| -1 | $\frac{-1+4}{16} = \frac{3}{16}$ | $-\frac{3}{16}$ | $\frac{3}{16}$ |
| 1 | $\frac{1+4}{16} = \frac{5}{16}$ | $\frac{5}{16}$ | $\frac{5}{16}$ |
| 2 | $\frac{6}{16}$ | $\frac{12}{16}$ | $\frac{24}{16}$ |

$\sum x \cdot f = \frac{5}{8}$ $\sum x^2 \cdot f = \frac{5}{2}$

$\therefore \sum f = 1 \rightarrow \frac{2}{16} + \frac{m+4}{16} + \frac{5}{16} + \frac{6}{16} = 1$

$\rightarrow \frac{2+m+4+5+6}{16} = 1$

$\rightarrow \frac{m+17}{16} = 1$

$\rightarrow m + 17 = 16 \rightarrow m = -1$

$\mu = \sum x \cdot f = \frac{5}{8}$

$\sigma^2 = \sum x^2 \cdot f - (\mu)^2 = \frac{5}{2} - \left(\frac{5}{8}\right)^2 = \frac{135}{64}$

4 A box contains 6 cards ; 2 cards carry the number two , 3 cards carry the number 3 and a card carries the number 11 , if a card has been randomly drawn and the discrete random variable X is defined as " the appeared number on the drawn card "

Find :

- a) The probability distribution function of the variable x .
- b) Expectation and standard deviation of the variable x .
- c) The coefficient of variation.

Solution

$x \in \{2, 3, 11\}$ "الإرقام الى على الكروت هي x"

$$f(2) = \frac{2}{6} = \frac{1}{3}$$

$$f(3) = \frac{1}{2}, \quad f(11) = \frac{1}{6}$$

لا حظ : عدد الكروت المطلوبه / عدد كل الكروت

| x_r | $F(x_r)$ | $X_r \cdot f(x_r)$ | $x_r^2 \cdot f(x_r)$ |
|--------------|---------------|--------------------|----------------------|
| 2 | $\frac{2}{6}$ | $\frac{4}{6}$ | $\frac{8}{6}$ |
| 3 | $\frac{3}{6}$ | $\frac{9}{6}$ | $\frac{27}{6}$ |
| 11 | $\frac{1}{6}$ | $\frac{11}{6}$ | $\frac{121}{6}$ |
| Total | | 4 | 26 |

a) $\mu = \sum x_r \cdot f = 4$

b) $\sigma^2 = \sum x_r^2 \cdot f - (\mu)^2$
 $= 26 - (4)^2 = 10$

c) $C = \left(\frac{\sigma}{\mu} \times 100\right) \%$
 $= 79 \%$

5 Choose the correct answer :

1) if the probability distribution of the discrete random variable X is { (0, 0.25) , (1, 0.5) , (2, 0.25) } then its expectation equals :

- a) 0.5 b) 1 c) 1.25 d) 1.5

2) If X is a discrete random variable and the expectation equals

$0.6, \sum_{r=1}^n x_r^2 \cdot f(x_r) = 4.36$ then it's standard deviation equals:

- a) 1.94 b) 2 c) 3.76 d) 4

3) if x is a discrete random variable and the expectation equals

$0.4, \sum_{r=1}^n x_r^2 \cdot f(x_r) = 6.16$ then it's variance equals

- a) 2.4 b) 5.76 c) 6 d) 6.56

Solution

1)

| | | | |
|------|------|-----|------|
| X | 0 | 1 | 2 |
| F(x) | 0.25 | 0.5 | 0.25 |
| x.f | 0 | 0.5 | 0.5 |

$M = \sum x \cdot f = 0 + 0.5 + 0.5 = 1$

b) 1

2) $\mu = 0.6, \sum x^2 \cdot f = 4.36$

$\therefore \sigma^2 = \sum x^2 \cdot f - (\mu)^2$
 $= 4$

$\therefore \sigma = \sqrt{4} = 2$

Standard Deviation b) 2

3) $\sigma^2 = 6.16 - (0.4)^2 = 6$
 Variance

c) 6

6 If x is a continuous random variable whose probability density function is :

$$f(x) = \begin{cases} \frac{1}{6}(2x - 1), & 1 \leq x \leq 3 \\ \text{Zero} , & \text{Otherwise} \end{cases}$$

- a) Prove That : F(x) is probability density function
- b) Find $p(X \leq 2), p(X > 2.5), p(2 \leq X \leq 2.5)$

Solution

a) $p(1 < x < 3) = \frac{1}{2} [f(1) + f(3)] \times (3 - 1)$

*We get

$F(1) = \frac{1}{6}(2 \times 1 - 1) = \frac{1}{6}$

$F(3) = \frac{1}{6}(2 \times 3 - 1) = \frac{5}{6}$

$\therefore p(1 < x < 3) = \frac{1}{2} \left[\frac{1}{6} + \frac{5}{6} \right] \times (2) = 1$ "proof الـ"

b) $p(x \leq 2) = p(1 \leq x \leq 2)$ $p(x \leq 2.5) = p(2.5 \leq x \leq 3)$
 $= \frac{1}{2} [f(1) + f(2)] \times (2 - 1)$ $= \frac{1}{2} [f(2.5) + f(3)] \times (3 - 2.5)$

$= \frac{1}{2} \left[\frac{1}{6} + \frac{3}{6} \right] \times (1) = \frac{1}{3}$ $= \frac{1}{2} \left[\frac{4}{6} + \frac{5}{6} \right] \times (0.5) = \frac{3}{8}$

$p(2 < x < 2.5) = \dots \rightarrow = \frac{7}{24}$

7 If x is a continuous random variable whose probability density function is :

$$f(x) = \begin{cases} \frac{2x + k}{24}, & 1 < x < 4 \\ \text{Zero} , & \text{Otherwise} \end{cases}$$

- a) Find the value of k
- b) Find $p(X > 3)$

Solution

a) We know that

$p(1 < x < 4) = 1$

$\therefore \frac{1}{2} [f(1) + f(4)] \times (4 - 1) = 1$

علشان تجيب f(1) عوض فوق عند x = 1

$\therefore \frac{1}{2} \left[\frac{2+k}{24} + \frac{8+k}{24} \right] \times (3) = 1$

$\therefore \frac{3}{2} \left[\frac{2+k+8+k}{24} \right] = 1$

$\frac{10 + 2k}{24} = \frac{2}{3}$

$10 + 2k = 16 \rightarrow k = 3$

b) $p(x > 3) = p(3 < x < 4)$

$= \frac{1}{2} [f(3) + f(4)] \times (4 - 3)$

$= \frac{1}{2} \left[\frac{9}{24} + \frac{11}{24} \right] \times (1) = \frac{5}{12}$

8

If x is a continuous random variable whose probability density function is :

$$f(x) = \begin{cases} \frac{1}{32}(x+4) & \text{where } -k \leq x \leq k \\ \text{Zero} & , \text{ Otherwise} \end{cases}$$

Find:-

- 1) The value of K 2) $p(x \leq 0)$ 3) $p(-2 \leq x \leq 2)$

Solution

1) We know that

$$p(-k \leq x \leq k) = 1$$

$$\therefore \frac{1}{2} [f(-k) + f(k)](k - (-k)) = 1$$

$$f(-k) = \frac{1}{32}(-k+4)$$

$$f(k) = \frac{1}{32}(k+4)$$

$$\therefore \frac{1}{2} \left[\frac{1}{32}(-k+4) + \frac{1}{32}(k+4) \right] (2k) = 1$$

$$\rightarrow \left[-\frac{1}{32}k + \frac{4}{32} + \frac{1}{32}k + \frac{4}{32} \right] \times k$$

$$\rightarrow \frac{8}{32k} = 1 \rightarrow \boxed{k=4}$$

- 2) $p(x < 0) = p(-4 < x < 0)$ 3) $p(-2 < x < 2) = \dots$

Chapter 4 :

First : Normal Distribution

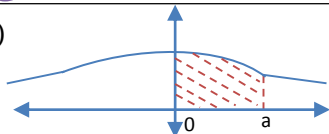
*ركز كده ... لما هكتب P(a) بيفي انا قصدي دي الي هنجيب منها الجدول ... نفس الكلام لو P(b) .

First :-

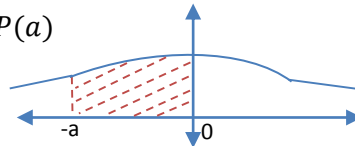
Rules

I

1) $P(0 \leq x \leq a) = P(a)$

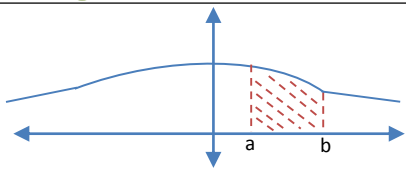


2) $P(-a \leq x \leq 0) = P(a)$

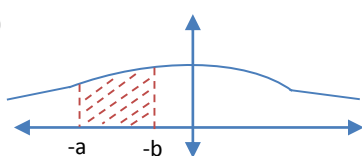


II

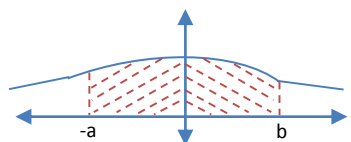
1) $P(a \leq x \leq b)$
 $= P(b) - P(a)$



2) $P(-a \leq x \leq -b)$
 $= P(a) - P(b)$

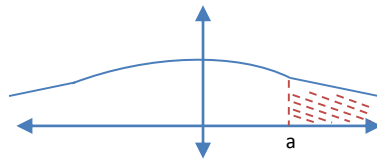


3) $P(-a \leq x \leq b)$
 $= P(a) + P(b)$

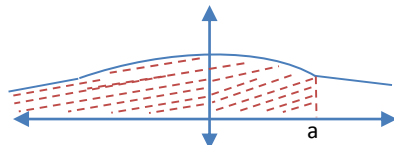


III

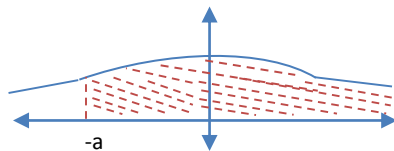
1) $P(x \geq a) = \frac{1}{2} - P(a)$



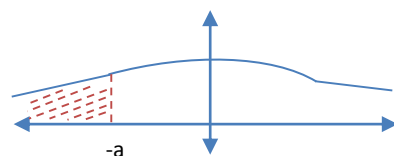
2) $P(x \leq a) = \frac{1}{2} + P(a)$



3) $P(x \geq -a) = \frac{1}{2} + P(a)$



4) $P(x \leq -a) = \frac{1}{2} - P(a)$



Second : (μ, σ)

If $P(x > a)$ but a not Standard Normal Variable
 $\rightarrow \frac{a - \mu}{\sigma}$

لو جالك المسأله فيها μ, σ علي طول خد
 Coefficient of σ

Example : $P(x < \mu - 1.56) = P(x < -1.5)$
 $P(\mu - 1.96 \leq x \leq \mu + 1.26)$
 $= P(-1.9 \leq x \leq 1.2)$
 $P(\mu < x < \mu + \sigma) = P(0 < x < 1)$

Third : to Find K

I رقم معروف
 1) $P(0 \leq x \leq K) = m$
 $\rightarrow P(K) = m$
 "هات ال K من الجدول"
 2) $P(-K \leq x \leq 0) = m$
 $\rightarrow P(K) = m$

II
 1) $P(x \geq K)$ $\left\{ \begin{array}{l} P(K) = m - 0.5 \\ K \text{ is } -ve \\ P(K) = 0.5 - m \\ K \text{ is } +ve \end{array} \right.$
 لو $m > \frac{1}{2}$
 لو $m < \frac{1}{2}$

2) $P(x \leq K)$ $\left\{ \begin{array}{l} P(K) = m - 0.5 \\ K \text{ is } +ve \\ P(K) = 0.5 - m \\ K \text{ is } -ve \end{array} \right.$
 لو $m > \frac{1}{2}$
 لو $m < \frac{1}{2}$

First

1) $P(0 \leq x \leq 0.63)$
 solution
 $P(0 \leq x \leq 0.63) = P(0.63)$

| | | | | |
|-----|------|-----|--------|-----|
| 0.0 | 0.01 | 0.2 | 0.3 | ... |
| 0.1 | | | | ... |
| ... | | | | ... |
| 0.6 | | | 0.2357 | |

$\therefore P(0 \leq x \leq 0.63) = 0.2357$
 "بص علي الجدول"

2) $P(z \leq 1.4)$
 solution
 $= 0.5 + P(1.4)$
 $= 0.5 + 0.4192$
 $= 0.9192$

3) $P(z \geq 0.95)$
 solution
 $= 0.5 + P(0.95)$
 $= 0.5 + 0.3289$
 $= 0.1711$

4) $P(-1.25 \leq x \leq 0)$
 solution
 $= P(1.25)$
 $= 0.3944$

5) $P(z \leq -1.6)$
 solution
 $= 0.5 - P(1.6)$
 $= 0.5 - 0.4452$
 $= 0.0548$

6) $P(z \geq -2.32)$
 solution
 $= 0.5 + P(2.32)$
 $= 0.5 + 0.4898$
 $= 0.9898$

7) $P(-1.4 \leq z < 1.4)$
 solution
 $= P(1.4) + P(1.4)$
 $= 0.4192 + 0.4192$
 $= 0.8384$

8) $P(-0.7 \leq x \leq 2.4)$
 solution
 $= P(0.7) + P(2.4)$
 $= 0.2580 + 0.4918$
 $= 0.7498$

9) $P(0.4 \leq z \leq 1.6)$
 solution
 $= P(1.6) - P(0.4)$
 $0.4452 - 0.1554$
 $= 0.2898$

10) $P(-1.4 < z < -0.34)$
 solution
 $= P(1.4) - P(0.34)$
 $0.4192 - 0.1331$
 $= 0.2861$

Second

1 If X is a normal random variable whose mean is μ and standard deviation is σ , Find :-

- a) $P(X > \mu - 1.5\sigma)$
- b) $P(X < \mu - 0.5\sigma)$
- c) $P(\mu - 1.96\sigma < X < \mu + 1.96\sigma)$

solution

| | |
|--|--|
| <p>a) $P(X > -1.5)$ $= 0.5 + P(1.5)$ $= 0.5 + 0.4332$ $= 0.9332$</p> | <p>b) $P(X < -0.5)$ $= 0.5 + P(0.5)$ $= 0.5 + 0.1915$ $= 0.3085$</p> |
| <p>c) $P(-1.96 < x < 1.96)$ $= P(1.96) \times 2$ $= 0.4750 \times 2$ $= 0.95$</p> | |

2 If X is a normal random variable whose mean is μ and standard deviation is σ , Find :-

- a) if: $P(x \geq 180) = 0.0062$, $\mu = 165$ calculate σ
- b) if: $P(x > 35) = 0.8643$, $\sigma = 5$ calculate μ
- c) if: $P(x \leq 170) = 0.0228$, $\sigma = 7$ calculate μ
- d) if: $P(x \leq K) = 0.8944$, $\mu = 125$, $\sigma = 8$ calculate K
- e) if: $P(x > K) = 0.9452$, $\mu = 50$, $\sigma = 5$ calculate K

solution

| | |
|---|--|
| <p>a) Let $P(x \geq K) = 0.0062$ $\therefore P(K) = 0.5 - 0.0062$ $= 0.4938$ "هندور في الجدول" $\therefore K = 2.5$ $\therefore 2.5 = \frac{180 - \mu}{\sigma}$ $\therefore \sigma = 6$</p> | <p>b) Let $P(x > K) = 0.8643$ $\therefore P(K) = 0.8643 - 0.5$ $= 0.3643$ $\therefore K = -1.1$ $\therefore \frac{35 - \mu}{5} = -1.1$ $\therefore \mu = 40.5$</p> |
|---|--|

| | |
|---|--|
| <p>c) Let $P(x \leq K) = 0.0228$ $\therefore P(K) = 0.5 - 0.228$ $= 0.4772$ $\therefore K = -2$ $\therefore \frac{170 - \mu}{7} = -2$ $\therefore \mu = 184$</p> | <p>d) $P(x \leq K) = 0.8944$ $\therefore P(K) = 0.8944 - 0.5$ $= 0.3944$ $\therefore K = 1.25$ $\therefore \frac{K - 125}{8} = 1.25$ $\therefore K = 135$</p> |
|---|--|

e) $P(x \leq K) = 0.9452$
 $\therefore P(K) = 0.9452 - 0.5$
 $= 0.4452$
 $\therefore K = -1.6$
 $\therefore \frac{K - 50}{5} = -1.6$
 $\therefore K = 42$

3 $P(-0.44 \leq z \leq K) = 0.5588$

solution

$P(0.44) + P(K) = 0.5588$
 $\therefore 0.1700 + P(K) = 0.5588$
 $P(K) = 0.3888$
 $\therefore K = 1.22$

4 $P(K \leq z \leq 2.1) = 0.2906$

solution

$\therefore P(2.1) = 0.4821$ " > 0.2906 "
 $\therefore P(K \leq z \leq 2.1)$
 $= P(2.1) - P(K) = 0.2906$
 $\therefore 0.4821 - P(K) = 0.2906$
 $\therefore P(K) = 0.1915$
 $\therefore K = 0.5$

Third

1 If the marks of the Students at a school are a normal random variable whose mean $\mu = 44$ and its standard deviation is σ where 22.66% of students have got more than 50 marks. Find the value of σ

solution

Let X be the normal random variable expressing the marks of the students

$\therefore P(X > 50) = \frac{22.26}{100}$
 $\therefore P\left(Z > \frac{50 - 44}{\sigma}\right) = 0.2266$

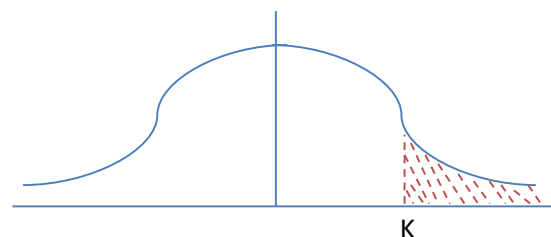
$\therefore P(Z > K) = 0.2266$ Where $K = \frac{6}{\sigma}$. $K > 0$

$\therefore P(0 \leq z < K) = 0.5 - 0.2266 = 0.2734$

$\therefore K = 0.75$

$\therefore \frac{6}{\sigma} = 0.72$

$\therefore \sigma = \frac{6}{0.75} = 8$



- 2 In a math exam, the marks of the students have been followed a normal distribution whose mean is 70 and its standard deviation is 5. Calculate the number of students whose marks are probably greater than 78 if it is known that the number of students applied to this exam is 10000.

solution

$$\mu = 70$$

$$\sigma = 5$$

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$$P(x > 78) = P\left(x > \frac{78 - 70}{5}\right)$$

$$= P(x > 1.6)$$

$$= 0.5 - P(1.6)$$

$$= 0.5 - 0.4452$$

$$= 0.0548$$

and the no. of students = 0.0548×10000

$$= 548$$

- 3 If the wages of the workers at a factory follow a normal distribution whose mean $\mu = 75$ LE and its standard deviation $\sigma = 10$, Find :-

- The Percentage of the workers whose wages are more than 90 LE
- The percentage of the workers whose wages are less than 55 LE
- The Percentage of the workers whose wages range between 60 LE and 80 LE

solution

$$\begin{aligned} \text{a) } \therefore P(X > 90) &= P\left(Z > \frac{90-75}{10}\right) \\ &= 0.5 - P(0 \leq Z \leq 1.5) = 0.5 - 0.4332 = 0.0668 \end{aligned}$$

\therefore The percentage of the workers whose wages are more than 90 LE = **6.68%**

$$\begin{aligned} \text{b) } \therefore P(X < 55) &= P\left(Z < \frac{55-75}{10}\right) = P(Z < -2) \\ &= 0.5 - P(0 \leq Z \leq 2) = 0.5 - 0.4772 = 0.0228 \end{aligned}$$

\therefore The percentage of the workers whose wages are less than 55 LE = **2.28%** of the total Number

$$\begin{aligned} \text{c) } \therefore P(60 \leq Z \leq 80) &= P\left(\frac{60-75}{10} \leq Z \leq \frac{80-75}{10}\right) \\ &= P(-1.5 \leq Z \leq 0.5) = P(0 \leq Z \leq 1.5) + P(0 \leq Z \leq 0.5) \\ &= 0.4332 + 0.1915 = 0.6247 \end{aligned}$$

\therefore The percentage of the workers whose wages range between 60 LE and 80 LE = **62.47%** of the total number of the workers at the factory.