

شغف وفريقك خطوة بخطوة




شغف التعليمي
Educational passion

$$\begin{array}{l} 2 > -3 \\ 0.999... = 1 \\ \pi \approx 3.14 \\ \sqrt{2} \\ 1 + 2 \cdot 3 \\ 5(2 + 2) \\ 101_2 = 5_{10} \end{array}$$



القناة الرئيسية "فريق شغف التعليمي" 

<https://t.me/alsh276>

مكتبة شغف "بوت الملفات" 

https://t.me/passion_study_bot

حساب نهاية متتالية $\lim_{n \rightarrow +\infty} U_n = l$
 حساب نهاية متتالية $\lim_{n \rightarrow +\infty} U_n = +\infty$

تارين

$$U_n = \frac{\cos 2n}{\sqrt{n}}$$

$$-1 \leq \cos 2n \leq 1$$

$$\frac{-1}{\sqrt{n}} \leq \frac{\cos 2n}{\sqrt{n}} \leq \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0 \Rightarrow \lim U_n = 0$$

مقارنة $U_n \leftarrow$ المقابلة

$$U_n = \cos\left(\frac{2n\pi}{3n+1}\right)$$

$$\lim_{n \rightarrow +\infty} \frac{2n\pi}{3n+1} = \frac{2\pi}{3} \Rightarrow \lim_{n \rightarrow +\infty} U_n = \cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

مقارنة $U_n \leftarrow$

$$U_n = \sin\left(\frac{n\pi}{2n+1}\right)$$

$$\lim_{n \rightarrow +\infty} \left(\frac{n\pi}{2n+1}\right) = \frac{\pi}{2} \Rightarrow \lim_{n \rightarrow +\infty} U_n = \sin \frac{\pi}{2} = 1$$

مقارنة $U_n \leftarrow$

$$U_n = \sqrt{n^2+n} - n - \frac{1}{2}$$

$$\lim U_n = (\infty - \infty) \text{ عام تعيين}$$

$$U_n = \sqrt{n^2+n} - \left(n + \frac{1}{2}\right) = \frac{\sqrt{n^2+n} + \left(n + \frac{1}{2}\right)}{\sqrt{n^2+n} + \left(n + \frac{1}{2}\right)} = \frac{n^2+n - \left(n^2+n + \frac{1}{4}\right)}{\sqrt{n^2+n} + n + \frac{1}{2}}$$

$$= \frac{-\frac{1}{4}}{\sqrt{n^2+n} + n + \frac{1}{2}} \Rightarrow \lim_{n \rightarrow \infty} U_n = 0$$

Subject:

$$\lim_{n \rightarrow +\infty} U_n = \frac{\infty}{\infty} \text{ غير متعين}$$

$$U_n = \frac{n(\sqrt{n+2} - \sqrt{n+1})}{\sqrt{n^3 + 3n + 2}} \cdot \frac{\sqrt{n+2} + \sqrt{n+1}}{\sqrt{n+2} + \sqrt{n+1}}$$

$$= \frac{n}{\sqrt{n^3 + 3n + 2}} = \frac{n}{n\sqrt{1 + \frac{3}{n} + \frac{2}{n^2}}} = \frac{1}{\sqrt{1 + \frac{3}{n} + \frac{2}{n^2}}}$$

$$\sqrt{n^2} = |n| \Rightarrow \lim_{n \rightarrow \infty} U_n = 0$$

$$U_n = \frac{n! - 2}{n!}$$

$$\lim_{n \rightarrow +\infty} U_n = \frac{\infty}{\infty} \text{ غير متعين}$$

$$\lim_{n \rightarrow +\infty} n! = +\infty \text{ غير متعين}$$

$$U_n = \frac{n!(1 - \frac{2}{n!})}{n!} = 1 - \frac{2}{n!} \Rightarrow \lim_{n \rightarrow +\infty} U_n = 1 - 0 = 1$$

$$U_n = \frac{5n - 3}{3n - 5}$$

$$\lim_{n \rightarrow +\infty} U_n = \frac{5}{3}$$

$$U_n = n - \frac{1}{n+1}$$

$$\lim_{n \rightarrow +\infty} U_n = +\infty \Rightarrow \text{غير متعين } U_n$$

$$U_n = \sqrt{\frac{4n-3}{n+1}}$$

$$\lim_{n \rightarrow +\infty} \frac{4n-3}{n+1} = 4 \Rightarrow \lim_{n \rightarrow +\infty} U_n = \sqrt{4} = 2 \Rightarrow \text{غير متعين } U_n$$

$$U_n = \sqrt{\frac{2n^2-1}{3n+1}}$$

$$\lim_{n \rightarrow +\infty} \frac{2n^2-1}{3n+1} = +\infty \Rightarrow \lim_{n \rightarrow +\infty} U_n = \sqrt{+\infty} = +\infty \Rightarrow \text{غير متعين } U_n$$

Subject :

1

$$u_n = n + 1 - \cos n$$

$$-1 < \cos n \leq 1$$

$$1 \geq -\cos n \geq -1$$

$$n + 2 \geq n + 1 - \cos n \geq n$$

$$n \leq u_n \leq n + 2$$

$$\lim_{n \rightarrow +\infty} (n) = \lim_{n \rightarrow +\infty} (n + 2) = +\infty \Rightarrow \lim_{n \rightarrow +\infty} u_n = +\infty$$

مستقيمة الإضافة \Rightarrow مستقيمة u_n

تمارين إضافية:
أوجد نهاية كل من المتواليات:

$$1. x_n = \frac{n^2 + 1}{n + 1}$$

$$2. y_n = \frac{x_n}{n}$$

$$3. w_n = x_n - n$$

$$4. t_n = \frac{y_n - 1}{w_n - 1}$$

$$1. \lim_{n \rightarrow \infty} x_n = +\infty$$

الحل:

$$2. \lim_{n \rightarrow +\infty} y_n = \left(\frac{\infty}{\infty}\right) \text{ C.E.}$$

$$y_n = \frac{x_n}{n} = \frac{n^2 + 1}{n(n+1)} = \frac{n^2 + 1}{n^2 + n} \Rightarrow \lim_{n \rightarrow +\infty} y_n = 1$$

$$3. \lim_{n \rightarrow \infty} w_n = (\infty - \infty) \text{ C.E.}$$

$$w_n = x_n - n = \frac{n^2 + 1}{n + 1} - n = \frac{n^2 + 1 - n(n+1)}{n + 1} = \frac{n^2 + 1 - n^2 - n}{n + 1} \\ = \frac{-n + 1}{n + 1} \Rightarrow \lim_{n \rightarrow +\infty} w_n = -1$$

$$4. \lim_{n \rightarrow +\infty} t_n = \frac{1 - 1}{-1 - 1} = 0$$

$$u_n = \frac{\sqrt{n} + 1}{n + 1} \quad \lim_{n \rightarrow \infty} u_n = \left(\frac{\infty}{\infty}\right) \text{ C.E.}$$

$$u_n = \frac{\sqrt{n} \left(1 + \frac{1}{\sqrt{n}}\right)}{\sqrt{n} \left(\sqrt{n} + \frac{1}{\sqrt{n}}\right)} = \frac{1 + \frac{1}{\sqrt{n}}}{\sqrt{n} + \frac{1}{\sqrt{n}}} \Rightarrow \lim_{n \rightarrow +\infty} u_n = 0$$

متقاربة u_n \Leftarrow

Subject:

$$u_n = \frac{3n - \sqrt{9n^2 + 1}}{\sqrt{n^2 + 5}}$$

$$\lim_{n \rightarrow +\infty} u_n = \frac{\infty}{\infty} \quad \text{C.E}$$

$$u_n = \frac{(3n - \sqrt{9n^2 + 1})(3n + \sqrt{9n^2 + 1})}{\sqrt{n^2 + 5}(3n + \sqrt{9n^2 + 1})} = \frac{-1}{\sqrt{n^2 + 5}(3n + \sqrt{9n^2 + 1})}$$

$$\Rightarrow \lim_{n \rightarrow \infty} u_n = 0 \Rightarrow \bar{\sigma}, \bar{\sigma}_n u_n$$

$$u_n = n^2 \left(\sqrt{2 + \frac{1}{n}} - \sqrt{2} \right)$$

$$\lim_{n \rightarrow +\infty} u_n = (\infty \times 0) \quad \text{عدم تعيين}$$

$$\cancel{u_n = n^2} \quad u_n = \frac{n^2 \left(\sqrt{2 + \frac{1}{n}} - \sqrt{2} \right) \left(\sqrt{2 + \frac{1}{n}} + \sqrt{2} \right)}{\sqrt{2 + \frac{1}{n}} + \sqrt{2}}$$

$$= \frac{n^2 \left(\frac{1}{n} \right)}{\sqrt{2 + \frac{1}{n}} + \sqrt{2}} = \frac{n}{\sqrt{2 + \frac{1}{n}} + \sqrt{2}} \Rightarrow \lim_{n \rightarrow \infty} u_n = +\infty$$

متباينة $u_n \leftarrow$

$$u_n = \frac{n}{4} + \frac{2n}{n^2 + 1}$$

$$\lim_{n \rightarrow +\infty} u_n = \frac{\infty}{\infty} \quad \text{C.E}$$

$$u_n = \frac{n(n^2 + 1) + 8n}{4(n^2 + 1)} = \frac{n^3 + n + 8n}{4n^2 + 4} = \frac{n^3 + 9n}{4n^2 + 4} \Rightarrow \lim_{n \rightarrow \infty} u_n = +\infty$$

متباينة $u_n \leftarrow$