التحليل التوافقي :!

← عاملي
← ترتيب
← توافق

* فكرة العاملي :!

$5! = 5 \times 4 \times 3 \times 2 \times 1$
$4! = 4 \times 3 \times 2 \times 1$

صغير ! . كبير = كبير!
والعكس صحيح

بر $n! = (n-1)(n-2)(n-3) \dots 1$

أحب كل ما يأتي

- $\frac{21!}{20!} = \frac{21 \times 20!}{20!} \Rightarrow = 21$

- $\frac{17!}{15!} = \frac{17 \times 16 \times 15!}{15!} \Rightarrow = 262$

- $\frac{6!}{|3!|^2} = \frac{6!}{3! \cdot 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \cdot 3 \times 2 \times 1} \Rightarrow = 20$

- $\frac{9!}{5! \times 4!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1} = \Rightarrow = 126$

$$\frac{6! - 5!}{5!} = \frac{6 \times 5! - 5!}{5!} = \frac{5!(6-1)}{5!} \Rightarrow = 5$$

$$\frac{1}{5!} - \frac{42}{7!} = \frac{1}{5!} - \frac{42}{7 \times 6 \times 5!} = \frac{1}{5!} - \frac{1}{5!} \Rightarrow = 0$$

$$\frac{6! + 7!}{2! \times 3! \times 4!} =$$

نفرق

$$\frac{6!}{2! \times 3! \times 4!} + \frac{7!}{2! \times 3! \times 4!} =$$

$$\frac{6 \times 5 \times 4!}{2! \times 3! \times 4!} + \frac{7 \times 6 \times 5 \times 4!}{2! \times 3! \times 4!}$$

$$\frac{6 \times 5}{2! \times 3!} + \frac{7 \times 6 \times 5}{2! \times 3!}$$

$$\frac{6 \times 5}{2 \times 1 \times 3 \times 2 \times 1} + \frac{7 \times 6 \times 5}{2 \times 1 \times 3 \times 2 \times 1}$$

$$\frac{5}{2} + \frac{35}{2} \Rightarrow = 20$$

$$\frac{(2n)! - (2n-1)!}{2(n!) - (n-1)!}$$

$$= \frac{(2n)! - (2n-1)!}{2(n)! - (n-1)!} = \frac{(2 \cdot \quad - (2n-1)!}{2(n)(n-1)! - (n-1)!}$$

$$= \frac{(2n-1)!}{-(n-1)!} = \frac{(2n-1)(2n-2)\ldots(n)(n-1)!}{(n-1)!}$$

تشبه مثلثة باسكال

$$\Rightarrow = (2n-1)(2n-2)\ldots(n)$$

- $\frac{(n-1)!}{n!} - \frac{n!}{(n+1)!}$

$$= \frac{(n-1)!}{n(n-1)!} - \frac{n!}{(n+1)(n)!}$$

$$= \frac{1}{n} - \frac{1}{(n+1)} = \frac{n+1-n}{n(n+1)} \Rightarrow = \frac{1}{n^2+n}$$

$\frac{(2n+1)!}{(2n-1)!}$

$$= \frac{(2n+1)(2n)(2n-1)!}{(2n-1)!} \Rightarrow = 4n^2 + 2n$$

$\frac{1}{n!} - \frac{1}{(n+1)!}$

P_n^r ترتيب $\neq$ ،

حيث $n \geq r$ ، $n, r \geq 1$

$$P_n^r = \frac{n!}{(n-r)!}$$

ex

- $P_{10}^3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} \Rightarrow = 720$

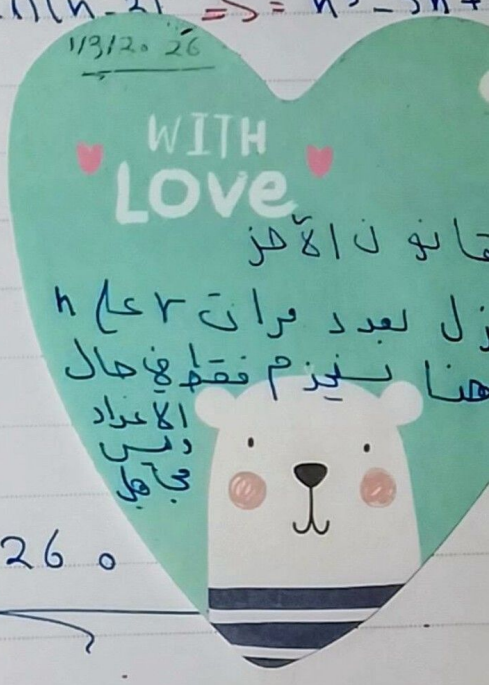
- $P_5^2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3!}{3!} \Rightarrow = 20$

- $P_n^3 = \frac{n!}{(n-3)!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = n(n-1)(n-2) \Rightarrow = n^3 - 3n^2 + \ldots$

- $\frac{P_6^2}{P_7^4} = \frac{6 \times 5}{7 \times 6 \times 5 \times 4} \Rightarrow = \frac{1}{28}$

- $P_3^2 \times P_4^3 = 3 \times 2 \times 7 \times 6 \times 5 \Rightarrow = 1260$

- $\frac{P_5^1}{P_8^3} = \frac{2}{8 \times 7 \times 6} \Rightarrow = \frac{1}{168}$



* حل معادلات ترتيب P_n^r

حلول وجد قيمة n

$\bullet P^4_{n+2} = 14 P^3_n$

1] نحدد شرط الحل

$\left.\begin{array}{l} n+2 \geqslant 4 \\ \rightarrow n \geqslant 2 \end{array}\right\} \left\{\begin{array}{l} n \geqslant 3 \\ \rightarrow n \geqslant 3 \end{array}\right.$

2] نأخذ العدد الأكبر $\Rightarrow n \geqslant 3$

3] نبدأ بحل المعادلة

$(n+2)(n+1)(n)(n-1) = 14(n)(n-1)(n-2)$

$(n+2)(n+1) = 14(n-2)$

$n^2+n+2n+2 = 14n - 28$

$n^2 - 11n + 30 = 0$

$(n-6)(n-5) = 0$

$\rightarrow n = 6$ ✓

$\rightarrow n = 5$ ✓

$\bullet P^4_n = 10 P^3_{n-1}$

$\left.\begin{array}{l} n \geqslant 4 \\ \rightarrow n \geqslant 4 \end{array}\right\} \left\{\begin{array}{l} n-1 \geqslant 3 \\ \rightarrow n \geqslant 4 \end{array}\right.$

$\Rightarrow n \geqslant 4$

$n(n-1)(n-2)(n-3) = 10(n-1)(n-2)(n-3)$

$\Rightarrow n = 10$

- $P_n^5 = 18 P_{n-2}^4$

$n \geq 5$, $n-2 \geq 4$

$\rightarrow n \geq 5$, $\rightarrow n \geq 6$

$\Rightarrow n \geq 6$

$n(n-1)(n-2)(n-3)(n-4) = 18(n-2)(n-3)(n-4)(n-5)$

$n^2 - n = 18n - 90$

$n^2 - 19n + 90 = 0$

$(n-10)(n-9)$

$n=10 \Rightarrow n = 10$ ✓

$n=9 \Rightarrow n = 9$ ✓

- $P_n^6 = 12 P_{n-1}^5$

$n \geq 6$, $n-1 \geq 5$

$\rightarrow n \geq 6$, $\rightarrow n \geq 6$

$\Rightarrow n \geq 6$

$n(n-1)(n-2)(n-3)(n-4)(n-5) = 12(n-1)(n-2)(n-3)(n-4)$

$\Rightarrow n = 12$ ✓

- $P^3_{n+1} = 2P^2_{n+2}$

$n+1 \geq 3$
$\rightarrow n \geq 2$

$n+2 \geq 2$
$n \geq 0$

$\Rightarrow n \geq 2$

$(n+1)(n)(n-1) = 2(n+2)(n+1)$

$n^2 - n = 2n + 4$
$n^2 - 3n - 4 = 0$
$(n-4)(n+1) = 0$
لما $\Rightarrow n = 4$
ولا $\Rightarrow n = -1$

- $2P^2_n + 50 = P^2_{2n}$

$n \geq 2$
$\rightarrow n \geq 2$

$2n \geq 2$
$\rightarrow n \geq 1$

$\Rightarrow n \geq 2$

$2(n)(n-1) + 50 = 2n(2n-1)$
$2(n^2 - 1) + 50 = 4n^2 - 2n$
$4n^2 - 2n^2 = 50$
$2n^2 = 50 \Rightarrow n^2 = 25$
لما $\rightarrow n = 5$
ولا $\rightarrow n = -5$ مرفوضة ✗

* التوافيق $C\binom{n}{r}$

حيث $n \geq r$

$$C\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

- $\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\cdot 2!} = \frac{5\times 4\times 3!}{3!\times 2\times 1} = 10$
- $\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!(4)!} = \frac{7\times 6\times 5\times 4!}{3\times 2\times 1\times 4!} = 35$

وأيضاً نكتب ان

$$C\binom{n}{r} = \frac{P_n^r}{r!}$$

- $\binom{5}{3} = \frac{5\times 4\times 3}{3\times 2\times 1} = 10$
- $\frac{\binom{4}{2}}{\binom{7}{3}} = \frac{\frac{4\times 3}{2\times 1}}{\frac{7\times 6\times 5}{3\times 2\times 1}} = \frac{2}{35}\cdot\frac{3}{} = \frac{6}{35}$
- $\frac{\binom{5}{3}\times\binom{6}{4}}{\binom{9}{3}} = \frac{\frac{5\times 4\times 3}{3\times 2\times 1} + \frac{6\times 5\times 4\times 3}{4\times 3\times 2\times 1}}{\frac{9\times 8\times 7}{3\times 2\times 1}} =$
- $\binom{6}{2} = \frac{6\times 5}{2\times 1} = 15$

* خواص التوافيق

- $\binom{n}{1} = n$
- $\binom{n}{0} = 1$ اصطلاحاً
- $\binom{n}{n} = 1$

. المتمم
كل توفيقة تساوي متمم لها

ex: $\binom{10}{3} = \binom{10}{7}$

ex: $\binom{5}{3} = \binom{5}{2}$

. أوجد قيمة n في كل مما يأتي

- $\binom{n}{2} = 36$ شرط $n \geq 2$

$$\frac{n!}{2!(n-2)!} = 36$$

$$\frac{n(n-1)(n-2)!}{2 \times 1 \times (n-2)!} = 36 \Rightarrow \frac{n^2 - n}{2} = 36 \Rightarrow n^2 - n - 72 = 0$$

$(n-9)(n+8) = 0$

$n = 9$ أو $n = -8$ مرفوض

• $2\binom{n+3}{2} = 3\binom{n+2}{3}$

$n+3 \geq 2$ $\rightarrow n \geq -1$ $\Big\}$ $\Big[$ $n+2 \geq 3$ $\rightarrow n \geq 1$

$\Rightarrow n \geq 1$

$$2\frac{(n+3)!}{2!(n+3-2)!} = 3\frac{(n+2)!}{3!(n+2-3)!}$$

$$2\frac{(n+3)(n+2)(n+1)!}{2\times 1(n+1)!} = 3\frac{(n+2)(n+1)n(n-1)!}{3\times 2\times 1(n-1)!}$$

$$2(n+3) = (n+1)n$$
$$2n+6 = n^2+n$$
$$0$$
$$(n-3)(n+2) = 0$$

$n = 3$ ✓

$n = -2$ ✗

• $3\binom{n}{4} = 14\binom{n}{2}$

$n \geq 4$ $\Big\}$ $n \geq 2$

$\rightarrow n \geq 4$ $\rightarrow n \geq 2$

$\Rightarrow n \geq 4$

$$3\frac{n!}{4!(n-4)!} = 14\frac{n!}{2!(n-2)!}$$

3 $\frac{n(n-1)(n-2)(n-3)(n-4)!}{4\times3\times2\times1(n-4)!} = \frac{4n(n-1)(n-2)!}{2\times1(n-2)!}$

3 $\frac{(n-2)(n-3)}{12} = 14$

$n^2 - 3n - 2n + 6 = 56$

$n^2 - 5n - 50 = 0$

$(n-10)(n+5) = 0$

ل ← $n = 10$ ✓

ل ← $n = -5$ ✗

• $2\binom{n}{2} = 72$

$n \neq 2$

$2\frac{n!}{2!(n-2)!} = 72$

$2\frac{n(n-1)(n-2)!}{2\times1(n-2)!} = 72$

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$(n-9)(n+8) = 0$

ل ← $n = 9$ ✓

ل ← $n = -8$ ✗

$\binom{10}{3n} = \binom{10}{n+2}$

$10 \geq 3n$ ⟹ $\frac{10}{3} \geq n$

$10 \geq n+2$ ⟹ $8 \geq n$

شرط الكل ⟹ $n \leq \frac{10}{3}$ ⟵ $n \in \{1, 2, 3\}$

هل يتساوى توافيق ؟

إذا كانت أحدهما متممة للأخرى

$3n + n + 2 = 10$

$4n = 8$

$\Rightarrow n = 2$ ✓

إذا كان $n = n$

$2 = 2$

$3n = n + 2$

$3n - n = 2$

$2n = 2$

$\Rightarrow n = 1$ ✓